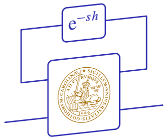
Introduction to Time-Delay Systems



lecture no. 5

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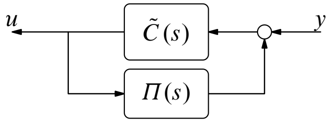
Outline

Dead-time compensation



Here

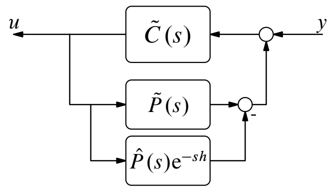
- $\tilde{C}(s)$ is primary controller (rational)
- $\Pi(s)$ is stable dead-time compensator of the form

$$\Pi(s) = \tilde{P}(s) - \hat{P}(s)e^{-sh}$$

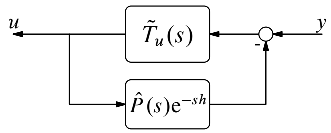
for some rational $\tilde{P}(s)$ and $\hat{P}(s)$.

Implementation of DTC: stable $\hat{P}$ case

In this case $\tilde{P}$ stable too[1] and controller can be safely implemented as



or even



where

$$\tilde{T}_u(s) := \tilde{C}(s)\big(I - \tilde{P}(s)\tilde{C}(s)\big)^{-1}$$

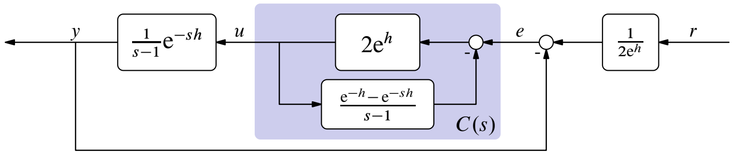
is (stable) controller sensitivity function. The only irrational element,

- e^{-sh} is a buffer,

which is easy to implement.

[1] Otherwise Π is unstable.

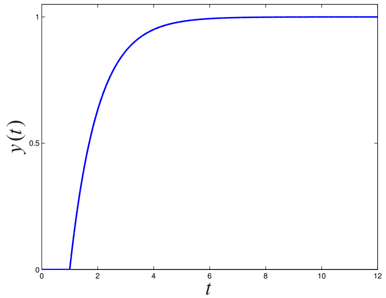
Unstable $\hat{P}$: example



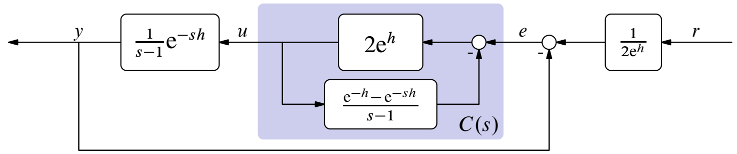
Closed-loop transfer function from r to y

$$T_r(s) = \frac{1}{s+1}\,\mathrm{e}^{-sh}$$

which results in step response (for $h = 1$):



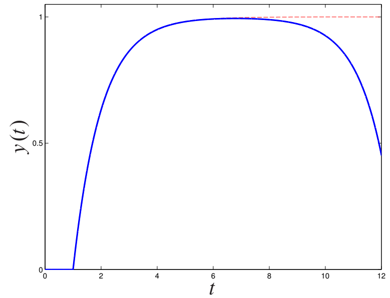
Unstable $\hat{P}$: example (contd)



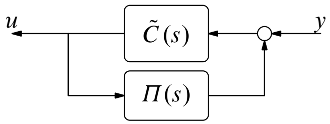
If DTC implemented as

$$\dot{\eta}(t) = \eta(t) + \mathrm{e}^{-h}u(t) - u(t-h),$$

closed loop is unstable (because of unstable pole/zero cancellations at $s = 1$):



Implementation with unstable $\hat{P}$



Main challenge:

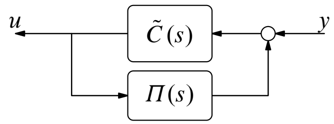
- avoid unstable pole/zero cancellation between implemented parts.

In other words, these cancellations should be performed

- analytically, within implementable block(s).

Conventionally, this is done within the DTC block, $\Pi(s)$.

Canceling unstable modes



Split components of Π to stable and anti-stable parts:

$$\tilde{P}(s) = \tilde{P}_{\mathrm{s}}(s) + \tilde{P}_{\mathrm{u}}(s) \quad \text{and} \quad \hat{P}(s) = \hat{P}_{\mathrm{s}}(s) + \hat{P}_{\mathrm{u}}(s)$$

(with strictly proper $\tilde{P}_{\mathrm{u}}(s)$ and $\hat{P}_{\mathrm{u}}(s)$, which is always possible). Then,

$$\Pi(s) = \underbrace{\tilde{P}_{\mathrm{s}}(s) - \hat{P}_{\mathrm{s}}(s)\mathrm{e}^{-sh}}_{\text{stable for every } \tilde{P}_{\mathrm{s}} \text{ and } \hat{P}_{\mathrm{s}}} + \underbrace{\tilde{P}_{\mathrm{u}}(s) - \hat{P}_{\mathrm{u}}(s)\mathrm{e}^{-sh}}_{\text{all poles must be canceled}}$$

Canceling unstable modes (contd)

Let

$$\hat{P}_{\mathrm{u}}(s) = \left[\begin{array}{c|c} A_{\mathrm{u}} & B_{\mathrm{u}} \\ \hline C_{\mathrm{u}} & 0 \end{array}\right]$$

be a *minimal* realization, then

$$\hat{P}_{\mathrm{u}}(s)\mathrm{e}^{-sh} = \left[\begin{array}{c|c} A_{\mathrm{u}} & B_{\mathrm{u}} \\ \hline C_{\mathrm{u}}\mathrm{e}^{-A_{\mathrm{u}}h} & 0 \end{array}\right] - \boldsymbol{\pi}_h \left\{ \left[\begin{array}{c|c} A_{\mathrm{u}} & B_{\mathrm{u}} \\ \hline C_{\mathrm{u}} & 0 \end{array}\right] \mathrm{e}^{-sh} \right\}$$

and, therefore, $\tilde{P}_{\mathrm{u}}(s) - \hat{P}_{\mathrm{u}}(s)\mathrm{e}^{-sh} \in H^\infty$ for some anti-stable $\tilde{P}_{\mathrm{u}}(s)$ iff

$$\tilde{P}_{\mathrm{u}}(s) = \left[\begin{array}{c|c} A_{\mathrm{u}} & B_{\mathrm{u}} \\ \hline C_{\mathrm{u}}\mathrm{e}^{-A_{\mathrm{u}}h} & 0 \end{array}\right].$$

In other words, the "unstable" part of $\Pi(s)$ is necessarily of the form

$$\tilde{P}_{\mathrm{u}}(s) - \hat{P}_{\mathrm{u}}(s)\mathrm{e}^{-sh} = \boldsymbol{\pi}_h\{\hat{P}_{\mathrm{u}}(s)\mathrm{e}^{-sh}\},$$

which is a distributed-delay element.

Distributed-delay element

Distributed-delay (DD) element can be expressed in the following forms:

$$\begin{aligned} \Pi(s) &= \boldsymbol{\pi}_h \left\{ \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \mathrm{e}^{-sh} \right\} \\ &= \left[\begin{array}{c|c} A & B \\ \hline C\mathrm{e}^{-Ah} & 0 \end{array}\right] - \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \mathrm{e}^{-sh} && \text{(Ä)} \\ &= \left[\begin{array}{c|c} A & \mathrm{e}^{-Ah}B \\ \hline C & 0 \end{array}\right] - \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \mathrm{e}^{-sh} && \text{(Å)} \\ &= C \int_0^h \mathrm{e}^{A(\theta-h)}\mathrm{e}^{-s\theta}\mathrm{d}\theta B && \text{(Ö)} \end{aligned}$$

DD element is

- entire function of s (no poles)
- FIR system (impulse response has support in $[0, h]$)

Implementing DD element

DD element is irrational (infinite-dimensional), so its

- precise implementation doesn't appear to be feasible.

Hence, approximations required.

Possibilities:

- incorporate resetting mechanism to avoid hidden modes to run away
- approximate distributed delay by lumped delays
- find finite-dimensional approximation, like Padé
  (complete pole/zero cancellation requirement imposes additional constraints)

Outline

Dead-time compensators and their implementation: general observations

Implementing DD elements via resetting mechanism

Implementing DD elements via lumped delay approximations

Respect internal loop: a case study

Form (Å) with zero initial conditions

The form (Å) can be implemented as

$$\begin{cases} \dot{x}(t) = Ax(t) + \big(\mathrm{e}^{-Ah}Bu(t) - Bu(t-h)\big) + \epsilon(t) \\ \eta(t) = Cx(t) \end{cases}$$

where $\epsilon(t)$ represents (inevitable) error in computing $\mathrm{e}^{-Ah}Bu(t) - Bu(t-h)$. Starting from zero initial conditions at $t = 0$ (i.e., $x(0) = 0$ and $u_\tau(0) \equiv 0$):

$$\begin{aligned} x(t) &= \int_0^t \mathrm{e}^{A(t-\theta)}\big(\mathrm{e}^{-Ah}Bu(\theta) - Bu(\theta-h) + \epsilon(\theta)\big)\mathrm{d}\theta \\ &= \int_0^t \mathrm{e}^{A(t-\theta-h)}Bu(\theta)\mathrm{d}\theta - \int_h^{\max\{t,h\}} \mathrm{e}^{A(t-\theta)}Bu(\theta-h)\mathrm{d}\theta \\ &\qquad + \underbrace{\int_0^t \mathrm{e}^{A(t-\theta)}\epsilon(\theta)\mathrm{d}\theta}_{x_\epsilon(t)} \end{aligned}$$

Form (Å) with zero initial conditions (contd)

Then

$$\begin{aligned} x(t) &= \int_0^t \mathrm{e}^{A(t-\theta-h)}Bu(\theta)\mathrm{d}\theta - \int_0^{\max\{t-h,0\}} \mathrm{e}^{A(t-\theta-h)}Bu(\theta)\mathrm{d}\theta + x_\epsilon(t) \\ &= \int_{\max\{t-h,0\}}^t \mathrm{e}^{A(t-\theta-h)}Bu(\theta)\mathrm{d}\theta + x_\epsilon(t) \\ &= \int_0^{\min\{t,h\}} \mathrm{e}^{A(\theta-h)}Bu(t-\theta)\mathrm{d}\theta + x_\epsilon(t) \\ &= \int_0^h \mathrm{e}^{A(\theta-h)}Bu(t-\theta)\mathrm{d}\theta \underbrace{- \int_{\min\{t,h\}}^h \mathrm{e}^{A(\theta-h)}Bu(t-\theta)\mathrm{d}\theta}_{x_{\mathrm{i.c.}}(t)} + x_\epsilon(t) \end{aligned}$$

and

$$\eta(t) = \underbrace{C\int_0^h \mathrm{e}^{A(\theta-h)}Bu(t-\theta)\mathrm{d}\theta}_{\text{expected output}} + \underbrace{Cx_{\mathrm{i.c.}}(t) + Cx_\epsilon(t)}_{\text{implementation errors}}.$$

Form (Å) with zero initial conditions: error analysis

$x_{\mathrm{i.c.}}$: we can write:

$$x_{\mathrm{i.c.}}(t) = \begin{cases} -\int_t^h \mathrm{e}^{A(\theta-h)}Bu(t-\theta)\mathrm{d}\theta & \text{if } t \in (0,h) \\ 0 & \text{otherwise} \end{cases}$$

starts from finite (might be large, if the true $u(t)$ is far from 0 in $t < 0$) and then vanishes after h time units (history accumulation period).

x_ϵ: this term

$$x_\epsilon(t) = \int_0^t \mathrm{e}^{A(t-\theta)}\epsilon(\theta)\mathrm{d}\theta$$

starts from $x_\epsilon(0) = 0$ and might diverge exponentially if A unstable.

Idea

Implement several systems of the form

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + \mathrm{e}^{-Ah}Bu(t) - Bu(t-h) \\ \eta_i(t) = Cx_i(t) \end{cases}$$

in parallel and make sure that

1. each system is periodically reset
   (so that its $x_\epsilon(t)$ is always bounded)
2. at every t at least one system has history accumulation stage completed
   (so that its $x_{\mathrm{i.c.}}(t) = 0$)

Implementation

It is sufficient to take two systems:

$$\dot{x}_1(t) = Ax_1(t) + \mathrm{e}^{-Ah}Bu(t) - Bu(t-h)$$

and

$$\dot{x}_2(t) = Ax_2(t) + \mathrm{e}^{-Ah}Bu(t) - Bu(t-h)$$

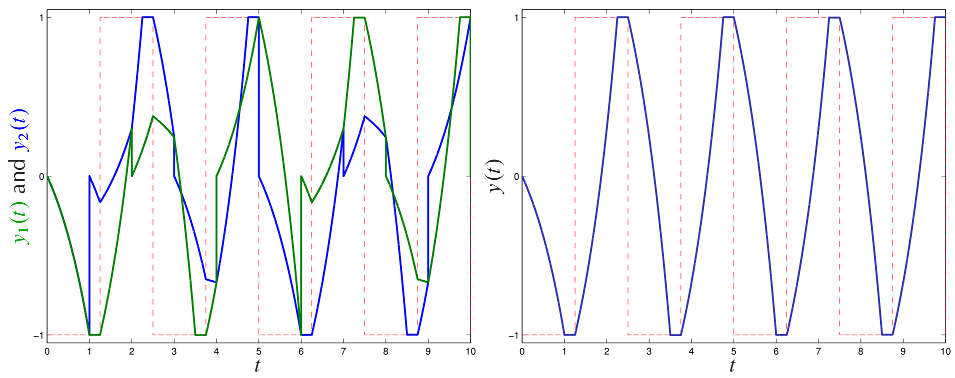
with

- reset mechanisms $x_1(2kh) = x_2((2k+1)h) = 0$ ($k \in \mathbb{Z}^+$)
- output formed according to

$$\eta(t) = \begin{cases} Cx_1(t) & \text{if } t \in [(2k+1)h, 2(k+1)h) \\ Cx_2(t) & \text{if } t \in [2kh, (2k+1)h) \end{cases}$$

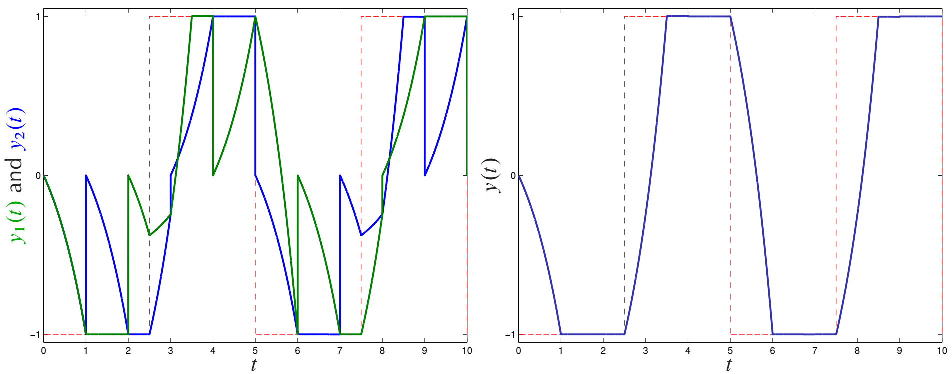
Example: $\Pi(s) = \frac{1}{1-\mathrm{e}^{-h}} \frac{\mathrm{e}^{-h}-\mathrm{e}^{-sh}}{s-1}$

Open-loop response of Π with $h = 1$ to square wave with period 2.5:



Example: $\Pi(s) = \frac{1}{1-\mathrm{e}^{-h}} \frac{\mathrm{e}^{-h}-\mathrm{e}^{-sh}}{s-1}$ (contd)

Open-loop response of Π with $h = 1$ to square wave with period 5:



Outros

Pros:

- easy to implement
- precision can be improved (by increasing number of systems)

Cons:

- nonlinear system
  (hard to analyze stability, hard to analyze performance, ...)

Outline

Dead-time compensators and their implementation: general observations

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Implementing DD elements via lumped delay approximations

Respect internal loop: a case study

Preliminary: numerical integration

Let $f(t)$ be integrable in $t \in [a, b]$. Then

$$\int_a^b f(t)\mathrm{d}t \approx \sum_{i=0}^{\nu} \alpha_i f\left(a + \tfrac{i}{\nu}(b-a)\right)$$

for some number of partitionings $\nu \in \mathbb{N}$ and some α_i (depend on method). Main steps:

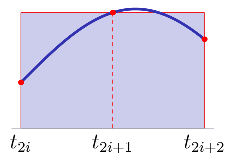
1. split $[a, b]$ into ν subintervals uniformly[2]
2. in each subinterval approximate $f(t)$ by function with calculable area
3. integral $\approx$ sum of approximation areas in each subinterval

Approximation

- performance improves as ν increases.

[2] Just for the sake of simplicity, intervals may be non-uniform.

Example: rectangle rule (ν even)



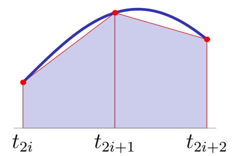
In this case

$$\alpha_i = \frac{b-a}{\nu}\begin{cases} 2 & \text{if } i \text{ odd} \\ 0 & \text{if } i \text{ even} \end{cases}$$

Approximation error (if $f'(t)$ and $f''(t)$ continuous and bounded):

$$\left|\int_a^b f(t)\mathrm{d}t - \sum_{i=0}^{\nu} \alpha_i f\left(a + \tfrac{i}{\nu}(b-a)\right)\right| \leq \frac{(b-a)^3}{6\nu^2} \max_{t\in[a,b]} \left|\frac{\mathrm{d}^2}{\mathrm{d}t^2} f(t)\right|$$

Example: trapezoid rule



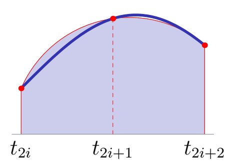
In this case

$$\alpha_i = \frac{b-a}{2\nu}\begin{cases} 2 & \text{if } i = 1, \ldots, \nu-1 \\ 1 & \text{if } i = 0, \nu \end{cases}$$

Approximation error (if $f'(t)$ and $f''(t)$ continuous and bounded):

$$\left|\int_a^b f(t)\mathrm{d}t - \sum_{i=0}^{\nu} \alpha_i f\left(a + \tfrac{i}{\nu}(b-a)\right)\right| \leq \frac{(b-a)^3}{12\nu^2} \max_{t\in[a,b]} \left|\frac{\mathrm{d}^2}{\mathrm{d}t^2} f(t)\right|$$

Example: Simpson's rule (ν even)



In this case

$$\alpha_i = \frac{b-a}{6\nu}\begin{cases} 1 & \text{if } i = 0, \nu \\ 2 & \text{if } i \text{ even but neither } 0 \text{ nor } \nu \\ 4 & \text{if } i \text{ odd} \end{cases}$$

Approximation error (if $f^{(i)}(t)$, $i = 1, 2, 3, 4$, continuous and bounded):

$$\left| \int_a^b f(t)\mathrm{d}t - \sum_{i=0}^{\nu} \alpha_i f\left(a + \tfrac{i}{\nu}(b-a)\right) \right| \le \frac{(b-a)^5}{720\nu^4} \max_{t\in[a,b]} \left| \frac{\mathrm{d}^4}{\mathrm{d}t^4} f(t) \right|$$

Lumped-delay approximations

Consider form (Ö) and deduce from it

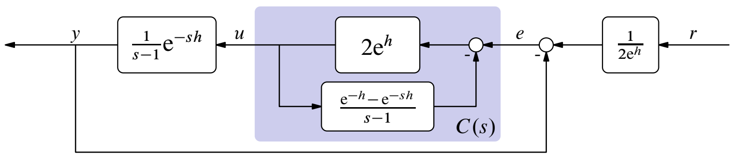
$$\Pi(s) = \int_0^h C\mathrm{e}^{A(\theta-h)} B \mathrm{e}^{-s\theta}\mathrm{d}\theta \approx \sum_{i=0}^{\nu} \alpha_i C\mathrm{e}^{A(i/\nu-1)h} B\mathrm{e}^{-shi/\nu} =: \Pi_\nu(s)$$

Π_ν is a lumped-delay system with entire and bounded transfer function, so

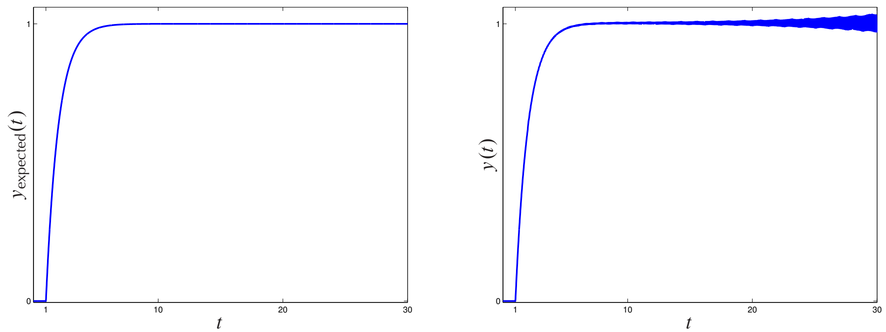
$$\Pi_\nu(s) \in H^\infty,$$

exactly what we need.

Example: unpleasant surprise



For trapezoid approximation and $\nu = 10$ we have unstable response:



Even more unpleasant is that system remains unstable as $\nu \to \infty$.

Approximation error

Let

$$\Delta_\Pi(s) := \Pi(s) - \Pi_\nu(s) \in H^\infty$$

be the approximation error. Its size may be measured as

$$\|\Delta_\Pi\|_\infty = \operatorname{ess\,sup}_{\omega\in\mathbb{R}} \|\Delta_\Pi(\mathrm{j}\omega)\|,$$

meaning that good match between Π and Π_ν requires

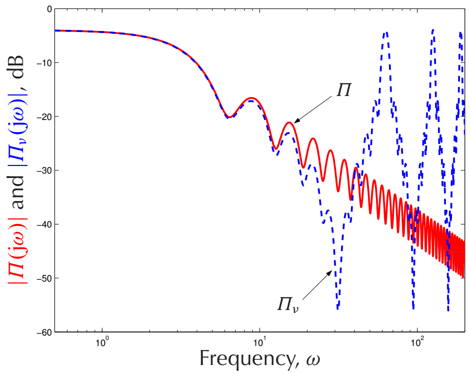
- accurate approximation over all frequencies.

At the same time, the derivative of

$$f(\theta) = C\mathrm{e}^{A(\theta-h)} B\mathrm{e}^{-\mathrm{j}\omega\theta}$$

unbounded (grows with ω), which questions the applicability of numerical integration methods to transfer functions.

Example: approximation error



As ν increases,

- mismatch just moves to higher frequencies

rather than vanishes[3].

[3] It can be shown that $\limsup_{\omega\to\infty}|\Delta_\Pi(\mathrm{j}\omega)| = \frac{h(1-\mathrm{e}^{-h})(\mathrm{e}^{h/\nu}+1)}{2\nu(\mathrm{e}^{h/\nu}-1)} \xrightarrow{\nu\to\infty} 1-\mathrm{e}^{-h}$.

Bad news

Key observations:

- $\Pi(s) = \tilde{P}(s) - \hat{P}(s)\mathrm{e}^{-sh}$ has a finite bandwidth

(as $\tilde{P}(s)$ and $\hat{P}(s)$ are strictly proper), whereas

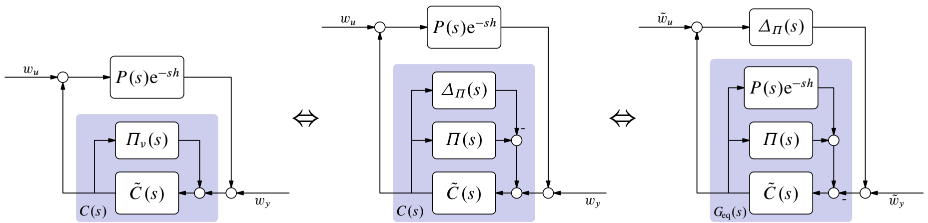
- $\Pi_\nu(s) = \sum_{i=0}^{\nu} \alpha_i C \mathrm{e}^{A(i/\nu-1)h} B\, \mathrm{e}^{-shi/\nu}$ has an infinite bandwidth.

In other words,

- Π_ν is an intrinsically poor approximation in the high-frequency range

Stability analysis

As $\Delta_\Pi \in H^\infty$, we may use loop shifting to separate Δ_Π from nominal parts:



We then get feedback connection of Δ_Π and

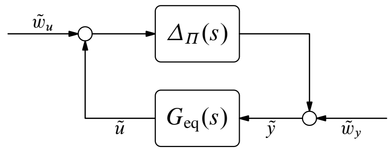
$$G_{\mathrm{eq}}(s) = -\tilde{C}(s)\big(I - (P(s)\mathrm{e}^{-sh} + \Pi(s))\tilde{C}(s)\big)^{-1}$$

If $\hat{P}(s) = P(s)$ (this is all what we saw by now),

$$G_{\mathrm{eq}}(s) = -\tilde{C}(s)\big(I - \tilde{P}(s)\tilde{C}(s)\big)^{-1} \in H^\infty$$

by design of $\tilde{C}$.

Stability analysis (contd)



By the Small Gain Theorem (SGT), this system is stable if

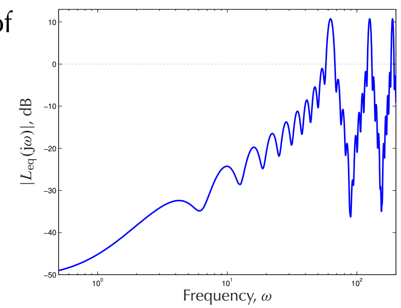
$$\|L_{\mathrm{eq}}(s)\|_\infty < 1, \qquad \text{where } L_{\mathrm{eq}}(s) := G_{\mathrm{eq}}(s)\Delta_\Pi(s)$$

For our example (trapezoid approximation of $\Pi(s) = \frac{\mathrm{e}^{-h}-\mathrm{e}^{-sh}}{s-1}$.)

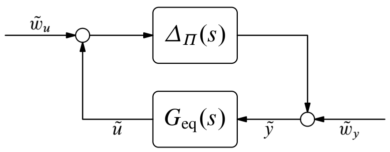
$$G_{\mathrm{eq}}(s) = 2\mathrm{e}^{h}\,\frac{s-1}{s+1}$$

and SGT condition fails. More important:

- it fails in high-frequency range



Stability analysis (contd)



We say that system is w-stable if

1. it is stable
2. $\limsup_{\omega\to\infty}\|L_{eq}(j\omega)\| < 1$

If system is not w-stable, it might be

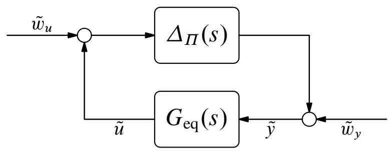
- destabilized by an arbitrarily small high-frequency mismatch,

which renders w-stability necessary for practical stability.

For system in our example $\limsup_{\omega\to\infty}|L_{eq}(j\omega)| = \frac{h/\nu(e^{h/\nu}+1)}{e^{h/\nu}-1}(e^h - 1) > 1$ (if $h = 1$ and $\nu = 10$), so that it is

- not w-stable.

Remedies



Two possibility:

- improve high-frequency robustness of $G_{eq}(s)$
  (always a good idea, regardless approximation precision, because

$$G_{eq} = -\tilde{C}(I - \tilde{P}\tilde{C})^{-1} = -C(I - Pe^{-sh}C)^{-1}$$

  is controller sensitivity, for which high high-frequency gain should be avoided)
- improve high-frequency precision of the lumped-delay approximation

Limiting approximations bandwidth

Since $\tilde{P}(s)$ and $\hat{P}(s)$ are strictly proper,

$$\begin{aligned}\Pi(s) &= \frac{\tau s+1}{\tau s+1}\boldsymbol{\pi}_h\{\hat{P}e^{-sh}\} \\ &= \frac{1}{\tau s+1}\left((\tau s+1)\left(\left[\begin{array}{c|c} A & B \\ \hline Ce^{-Ah} & 0\end{array}\right] - \left[\begin{array}{c|c} A & B \\ \hline C & 0\end{array}\right]e^{-sh}\right)\right) \\ &= \frac{1}{\tau s+1}\left(\tau Ce^{-Ah}B - \tau CBe^{-sh} + \boldsymbol{\pi}_h\left\{\left[\begin{array}{c|c} A & B \\ \hline C(I+\tau A) & 0\end{array}\right]e^{-sh}\right\}\right) \\ &= \frac{1}{\tau s+1}\Pi_\tau(s),\end{aligned}$$

where Π_τ is also a DD element. We may then

- approximate Π via approximating Π_τ,

which shall result in a finite-bandwidth approximation.

Limiting approximations bandwidth (contd)

Standard lumped-delay approximation of Π_τ is

$$\Pi_\tau(s) \approx \sum_{i=0}^{\nu}\alpha_i C(I+\tau A)e^{A(i/\nu-1)h}B\,e^{-shi/\nu}$$

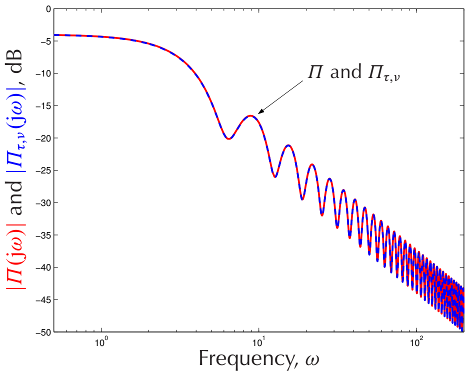
This results in

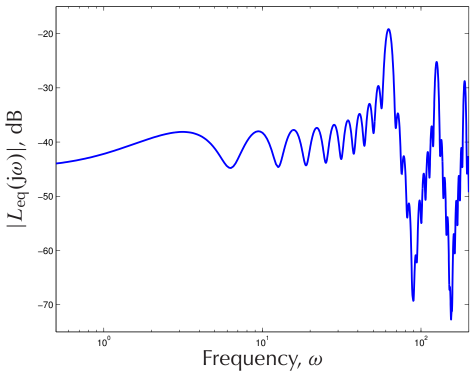
$$\Pi(s) \approx \frac{1}{\tau s+1}\sum_{i=0}^{\nu}\Pi_i\,e^{-shi/\nu} =: \Pi_{\tau,\nu}(s),$$

where

$$\Pi_i = \begin{cases} C(\alpha_0(I+\tau A)+\tau I)e^{-Ah}B & \text{if } i=0 \\ C(\alpha_\nu(I+\tau A)-\tau I)B & \text{if } i=\nu \\ \alpha_i C(I+\tau A)e^{A(i/\nu-1)h}B & \text{otherwise}\end{cases}$$

It works…





Step response is then virtually ideal.

Outros

Pros:

- precision can be improved (by increasing number of discr. steps ν)
- linear, so (relatively) easy to analyze

Cons:

- implementation cost grows with ν

Outline

Dead-time compensators and their implementation: general observations

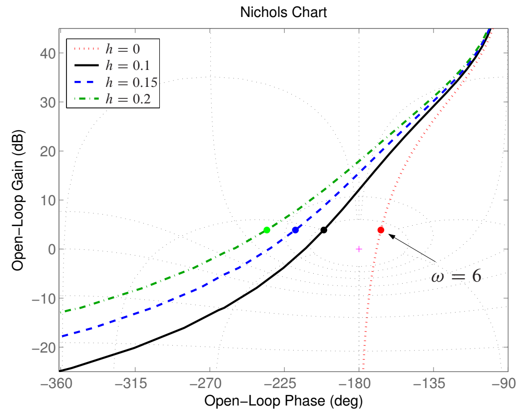
Implementing DD elements via resetting mechanism

Implementing DD elements via lumped delay approximations

Respect internal loop: a case study

Problem: servo for DC motor

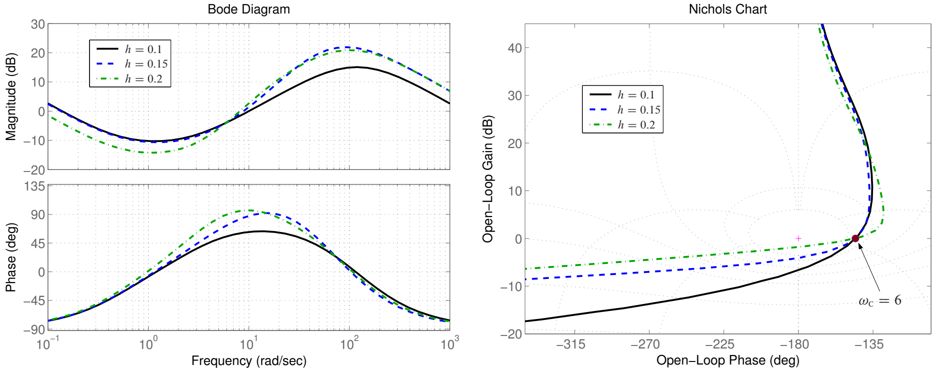
Plant: $P(s) = \dfrac{41.085}{s(0.71s+1)} e^{-sh}$ for $h \in \{0.1, 0.15, 0.2\}$



Specs:
1. integral action
2. (first) crossover $\omega_c = 6$rad/sec
3. phase margin $\mu_{ph} = 0.6$rad $\approx 34.4°$

Classical loop shaping

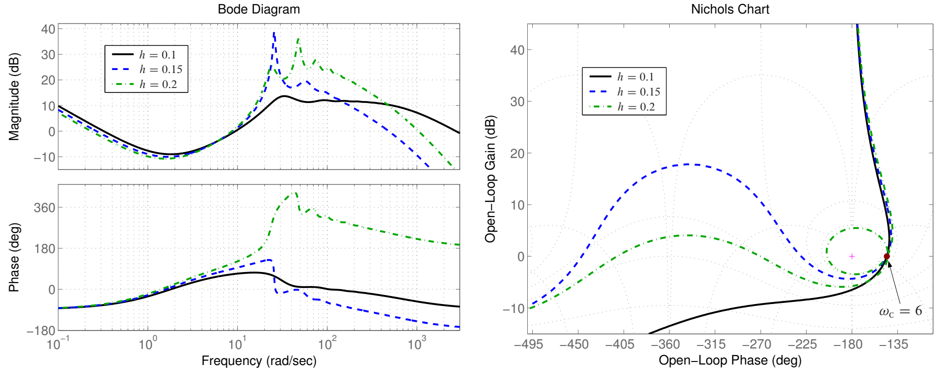
Delay makes it harder: with standard lead-lag elements[4] results not exciting



[4] The use of 2nd order leads and other loop shaping tricks might perhaps improve results, yet this is not a point here. . .

DTC-based design

Use of H^∞ loop shaping results in DTC controllers and resulting loops:



Off-the-shelf implementation

Let's use lumped-delay approximation with τ-augmentation and choose ν so that

$$\left|1 - \frac{\mu_{\mathrm{d,a}}}{\mu_{\mathrm{d}}}\right| \le 0.001,$$

where μ_{d} and $\mu_{\mathrm{d,a}}$ are delay margins of designed and implemented loops.

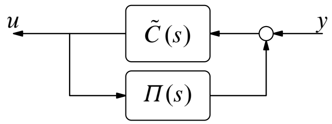
Results:

h	0.1	0.15	0.2
ν	7	435	5460

Scary:

- doubling delay increases ν by a factor of 780 !

Inaccuracy mechanisms: mind controller loop



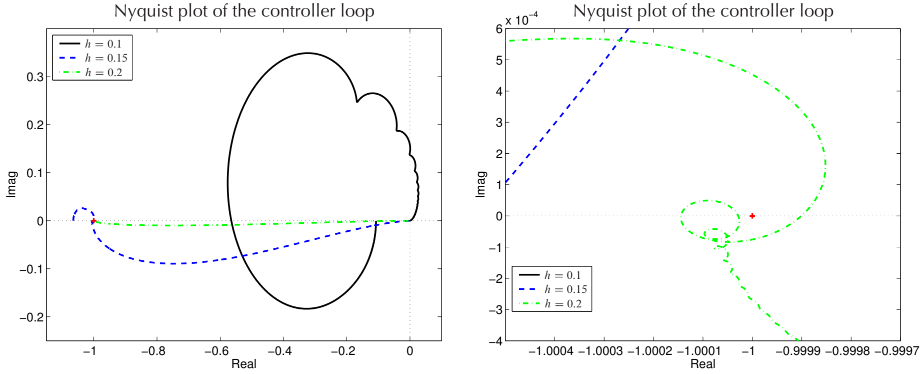
We implement controller as

- feedback interconnection of two systems, $\tilde{C}$ and Π.

It then makes sense to

- scrutinize this loop.

Inaccuracy mechanisms: mind controller loop (contd)



Clearly seen that

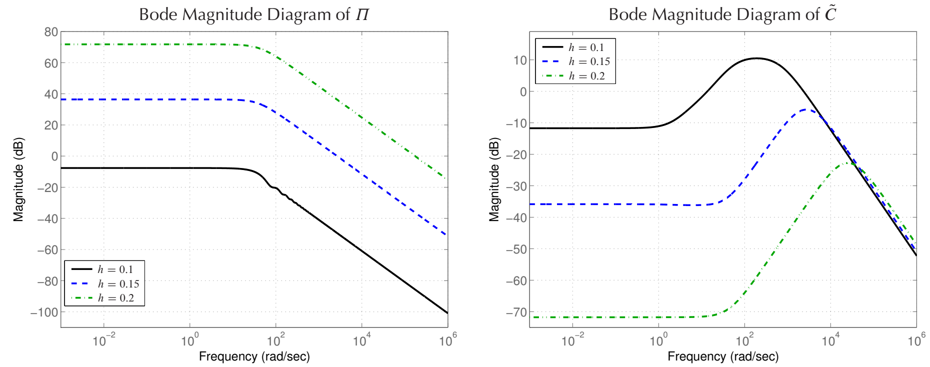
- as h increases stability margins decrease

which, in turn, makes controller loop

- extremely sensitive to numerical errors.

Inaccuracy mechanisms: loop disparity

Consider now each component of the controller internal loop:



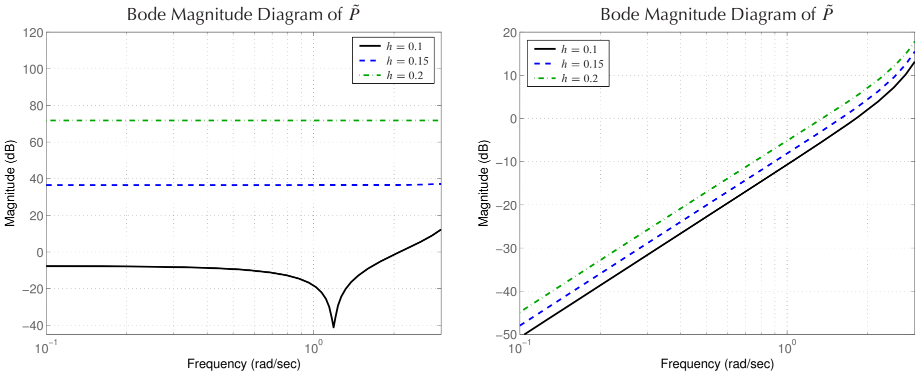
We see that as h increases

- $|\Pi(j\omega)|$ grows, whereas $|\tilde{C}(j\omega)|$ decreases

This results in an unbalanced loop and numerical errors in computing Π.

Inaccuracy mechanisms: loop disparity (contd)

Take a closer look at the components of $\Pi = \tilde{P} - \hat{P}e^{-sh}$:



Clearly,

$$\tilde{P}(s) = \left[\begin{array}{c|c} A & B \\ \hline Ce^{-Ah} & 0 \end{array}\right] \qquad \text{rather than} \qquad \hat{P}(s) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$$

is the component inflating Π and an apparent cause of this is the e^{-Ah} term.

Loop disparity: remedy

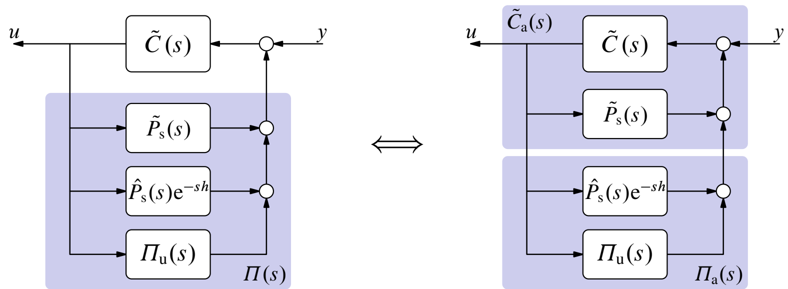
Two point to notice:

1. only stable modes of A cause problems via inflating e^{-Ah}
2. only unstable modes of A have to be canceled in DD element

This suggests split (we already saw it)

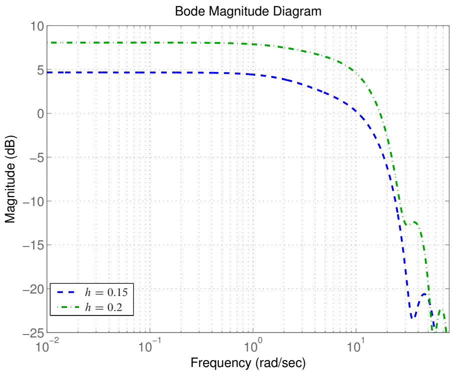
$$\Pi(s) = \Pi_s(s) + \Pi_u(s) =: \big(\tilde{P}_s(s) + \hat{P}_s(s)e^{-sh}\big) + \big(\tilde{P}_u(s) + \hat{P}_u(s)e^{-sh}\big)$$

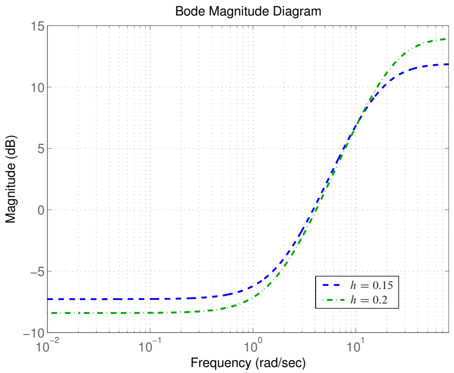
and the implementation via loop shifting:



(rational $\tilde{C}_a = \tilde{C}(I - \tilde{P}_s\tilde{C})^{-1}$ implemented as one piece).

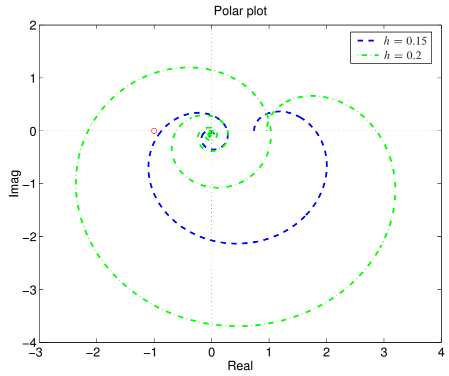
Loop shifting: components





More balanced. . .

Loop shifting: stability margins



Increased. . .

Loop shifting: results

h	0.15	0.2
$\nu_{\text{out of the box}}$	435	5460
$\nu_{\text{loop shifting}}$	25	52

Much better, isn't it?

Outros

Internal loop of controller

- must be respected

as its ill-posedness might cause implementation problems.

To understand underlying reasons of ill-posed loops and possible remedies,

- much yet to be done. . .