Introduction to Time-Delay Systems

Fall 2012

Homework no. 6

(submission deadline: 21.12.2012, 10:00am)

Problem 1 (40%). Consider the system (which is motivated by the dynamics of machining chatter):

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10-k & 10 & 0 & 0 \\ 5 & -15 & 0 & -0.25 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(t-h),$$

for $k \in \{0.25, 1\}$. For each of these two k's,

1. Calculate the precise delay margins (the first destabilizing delay h);
2. Calculate lower bounds on the delay margin using the Small Gain Theorem (or scaled Small Gain Theorem, if required). Try to maximize these lower bounds (in any case, the lower bound should not be less than a half of the precise delay margin).

Problem 2 (60%). Consider the distributed-delay system

$$\Pi(s) = \boldsymbol{\pi}_h \left\{ \frac{10s^2 + 2250s + 22500}{s^3 - 26s^2 + 2525s - 2500} \, \mathrm{e}^{-s} \right\}$$

and the task of its lumped-delay approximation $\Pi_{\mathrm{a}}(s)$ with coefficient chosen by the trapezoid integration rule. The approximation performance is measured by the quantity

$$\varepsilon := \frac{\|\Pi - \Pi_{\mathrm{a}}\|_\infty}{\|\Pi\|_\infty}.$$

1. Assuming a uniform split of the interval, i.e., an approximation of the form

   $$\Pi_{\mathrm{a}}(s) = \frac{1}{\tau s + 1} \sum_{i=0}^{\nu} \Pi_i \, \mathrm{e}^{-si/\nu}, \qquad \text{for some } \nu \in \mathbb{N} \text{ and constants } \Pi_i,$$

   try to determine τ for which $\varepsilon < 0.05$ is achieved by as small ν as possible.
2. Suggest a different lowpass filter choice *algorithm*, with which the approximation error of $\varepsilon < 0.05$ can be achieved with a smaller ν? Apply it to $\Pi(s)$ above.
3. Suggest a *algorithm* for a nonuniform partition of the delay interval $[0, 1]$ resulting in a smaller number of partitionings of the delay interval. Apply it to $\Pi(s)$ above.

In calculating norms for ε use sufficiently dense frequency grid in the range $[0.1, 10000]$, as well as the value of the static gain.