Introduction to Time-Delay Systems

Homework no. 5

(submission deadline: 5.12.2012, 8:00am)

Problem 1 (40%). Consider the system

$$\dot{x}(t) = Ax(t) + Bu(t - h) \tag{1}$$

and assume that we measure $x(kT)$, $k \in \mathbb{Z}$, and that $u(t) = \bar{u}_k$ for all $t \in [kT, (k + 1)T)$. In other words, we have a sampled-data system with the sampling period $T > 0$. Its stroboscopic model is

$$\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}_0\bar{u}_{k-l+1} + \bar{B}_1\bar{u}_{k-l}, \tag{2}$$

where $\bar{x}_k := x(kh)$, $l := \lceil h/T \rceil$, and

$$\bar{A} := \mathrm{e}^{AT}, \quad \bar{B}_0 := \int_0^{l-h/T} \mathrm{e}^{A\tau}\mathrm{d}\tau B, \quad \text{and} \quad \bar{B}_1 := \int_{l-h/T}^{h} \mathrm{e}^{A\tau}\mathrm{d}\tau B.$$

It can be shown that if some mild non-pathological sampling conditions hold, a discrete controller stabilizes (1) iff it stabilizes (2). Thus, to stabilize the hybrid system (1) we may design a stabilizing controller for the pure discrete system (2).

So your task is to design such controllers using two methods: a discrete counterpart of the Kwon-Pearson-Artstein reduction procedure and a discrete counterpart of the loop shifting procedure. No state augmentation is allowed, so all calculations should be in terms of matrices of the dimension of x.
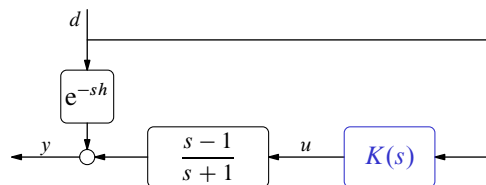

Figure 1: Feedforward control with measured disturbances

Problem 2 (40%). Consider the feedforward control scheme with measured disturbances depicted in Fig. 1. The goal is to design a stable K such that the error system

$$G_\mathrm{e}(s) = W(s)\Big(\mathrm{e}^{-sh} + \frac{s - 1}{s + 1}K(s)\Big)$$

is stable and its norm is minimal, where $W(s)$ is a weighting function, emphasizing frequencies where disturbance attenuation is important. Solve the problem for $W(s) = \frac{s+b}{s}$, $b > 0$, in both H^2 and H^∞ cases by finding the optimal K and the optimal achievable performance levels.

Problem 3 (20%). Can the H^2 optimal controller be in the form of the *classical Smith controller*? If yes, give an example of the problem resulting in such a controller.