Introduction to Time-Delay Systems

Fall 2012

Homework no. 4

(submission deadline: 28.11.2012, 8:00am)

Problem 1 (34%)**.** Consider the two-stage design of DTC-based controller for the plant $P(s) = \frac{1}{s}\mathrm{e}^{-sh}$ for $h > 0$. Propose a DTC element $\Pi(s)$ guaranteeing that

1. the order of the plant $\tilde{P}(s)$ used in $\boldsymbol{\mathcal{S}_1}$ is at most 2;
2. the static gain of the primary controller $\tilde{C}(s)$ designed in $\boldsymbol{\mathcal{S}_1}$ is preserved in $\boldsymbol{\mathcal{S}_2}$;
3. a crossover frequency $\omega_c > 0$ and the corresponding phase margin of the loop $\tilde{L}$ designed in $\boldsymbol{\mathcal{S}_1}$ are also the crossover frequency and phase margin of the system implemented in $\boldsymbol{\mathcal{S}_2}$.

For which h and ω_c these requirements can be met ?

Problem 2 (33%)**.** Consider the following distributed-delay system:

$$\Pi(s) = \int_0^h \mathrm{e}^{-(sI-A)\theta}\mathrm{d}\theta \in H^\infty$$

and its lumped-delay approximation

$$\Pi_\nu(s) = \frac{h}{\nu}\sum_{i=0}^{\nu-1}\mathrm{e}^{-(sI-A)ih/\nu} = \frac{h}{\nu}\sum_{i=0}^{\nu-1}\mathrm{e}^{Aih/\nu}\mathrm{e}^{-sih/\nu} \in H^\infty, \qquad \nu \in \mathbb{N}$$

which is motivated by the rectangle method of approximating definite integrals. Prove that

$$\|\Pi - \Pi_\nu\|_\infty \geq \frac{h}{\nu}\left\|(I - \mathrm{e}^{Ah/\nu})^{-1}(I - \mathrm{e}^{Ah})\right\| \qquad \text{and that} \qquad \lim_{\nu\to\infty}\|\Pi - \Pi_\nu\|_\infty \neq 0,$$

where $\|\cdot\|$ denotes the matrix spectral norm (the largest singular value).

Problem 3 (33%)**.** Consider the system in Fig. 1, for which the closed-loop transfer function from r to y is $\frac{1}{s+1}\mathrm{e}^{-s}$. Suggest and *justify* stable second- and fifth-order rational approximations $\Pi_r(s)$ of the DTC block $\Pi(s) = \frac{\mathrm{e}^{-1}-\mathrm{e}^{-s}}{s-1}$,

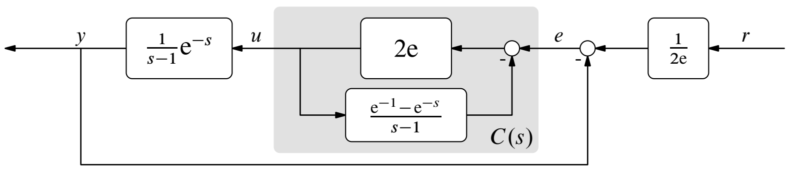

Figure 1: Dead-time compensator for unstable plant

which guarantee the *analytic* cancellation of the singularity at $s = 1$ (numerical canceling closely located pole and zero is not permitted). In each case, present the magnitude Bode plot of the approximation error $\Pi - \Pi_r$ and a Bode plot depicting the original controller $C(s)$ and its rational approximation. Calculate the resulting closed-loop transfer functions from r to y and present their step responses. Is the method extendible to DTCs with more than one pole/zero cancellations?