Introduction to Time-Delay Systems

Fall 2012

Homework no. 3

(submission deadline: 14.11.2012, 10:00am)

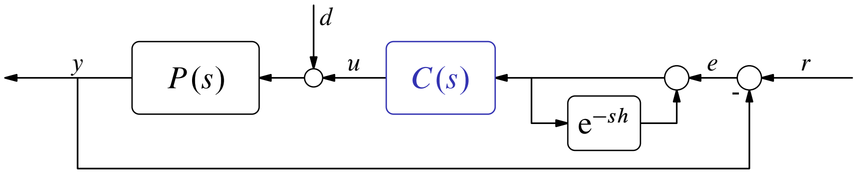

Figure 1: Repetitive control system

Problem 1 (20%)**.** Consider the repetitive control system depicted in Fig. 1. Assume that $P(s)$ is rational and strictly proper. Prove that there is no $C(s)$ that internally stabilizes this system.

Problem 2 (40%)**.** Consider the SISO system

$$\Sigma_h : \dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x(t-h) + \begin{bmatrix} b \\ 1 \end{bmatrix} u(t-h)$$

where $h > 0$.

1. How many poles in $\bar{\mathbb{C}}_0$ this system has?
2. Design a stabilizing controller that moves only unstable poles and works for every h. What closed-loop characteristic polynomial it yields? For what b and h is this possible?

Problem 3 (40%)**.** Let

$$\Sigma_h : \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11}x_1(t) + A_{12}x_2(t) + A_h x_2(t-h) + B_1 u(t) \\ A_{22}x_2(t) + B_2 u(t) \end{bmatrix}.$$

Find a control law based on measurements of $x_1(t)$ and $x_2(t)$ that stabilizes Σ_h (try to simplify it as much as possible). What are the stabilizability conditions? Is it possible that the system with $A_h = 0$ is not stabilizable, whereas the system becomes stabilizable for some A_h?