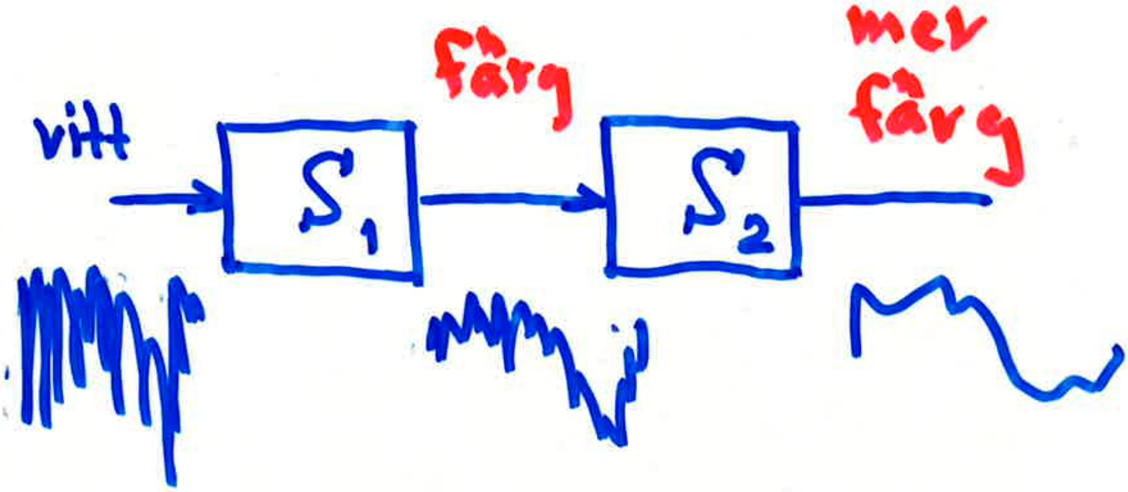
F4

Dagens idé:



Bara <u>en</u> bruskälla

SPEKTRALTÄTHET

KJÅ s 27

$$F(\omega) = F_a(\omega) + F_d(\omega) + F_s(\omega)$$

$F_a(\omega)$: $\int^{\omega} \phi(\omega')\,d\omega'$

$F_d(\omega)$: steg fkn $\sum_{\omega_\nu \leq \omega} F(\omega_\nu)$

$F_s(\omega)$: singular part konst nästan överallt

$\phi(\omega)$ Spektraltäthet

Kontinuerlig tid

$$\phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} r(t)\,dt$$

$$r(t) = \int_{-\infty}^{\infty} e^{i\omega t} \phi(\omega)\,d\omega$$

Diskret tid

$$\phi(\omega) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} r(n) e^{-in\omega}$$

$$r(n) = \int_{-\pi}^{\pi} e^{in\omega} \phi(\omega)\,d\omega$$

Exempel 1

$$\phi(\omega) = \frac{1.04 + 0.4\cos\omega}{1.25 + \cos\omega} = \frac{(e^{i\omega} + 0.2)(e^{-i\omega} + 0.2)}{(e^{i\omega} + 0.5)(e^{-i\omega} + 0.5)}$$

$$\phi(z) = \frac{(z+0.2)(z^{-1}+0.2)}{(z+0.5)(z^{-1}+0.5)} = H_1(z)\,H_1(z^{-1})$$

$$= \frac{1+0.2z}{z+0.5} \cdot \frac{1+0.2z^{-1}}{z^{-1}+0.5} = H_2(z)\,H_2(z^{-1})$$

$$= \frac{z+0.2}{1+0.5z} \cdot \frac{z^{-1}+0.2}{1+0.5z^{-1}} = H_3(z)\,H_3(z^{-1})$$

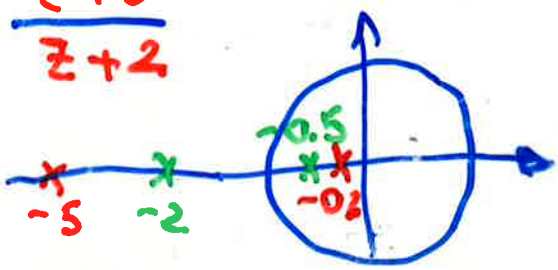
$$= \frac{1+0.2z}{1+0.5z} \cdot \frac{1+0.2z^{-1}}{1+0.5z^{-1}} = H_4(z)\,H_4(z^{-1})$$

$$H_1 = \frac{z+0.2}{z+0.5}$$

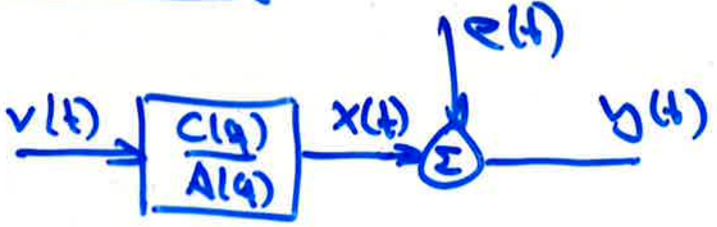
$$H_2 = \frac{1+0.2z}{z+0.5} = 0.2\,\frac{z+5}{z+0.5}$$

$$H_3 = \frac{z+0.2}{1+0.5z} = 2\,\frac{z+0.2}{z+2}$$

$$H_4 = \frac{1+0.2z}{1+0.5z} = 0.4\,\frac{z+5}{z+2}$$



Tolkning



e, v ober

$$\phi_y(z) = \lambda_e^2 + \lambda_v^2 \frac{CC^*}{AA^*} = \lambda_\varepsilon^2 \frac{DD^*}{AA^*}$$

$$\lambda_\varepsilon^2 DD^* = \lambda_v^2 CC^* + \lambda_e AA^*$$

$n+1$ ekv för $n+1$ obek.

Räcker med en bruskälla ε

Spektralfaktorering — Tillståndsmodell

$$x(t+1) = Ax(t) + v(t) \qquad |\lambda(A)| < 1$$
$$y(t) = Cx(t) + e(t)$$

$$E \begin{bmatrix} v \\ e \end{bmatrix} \begin{bmatrix} v^T & e^T \end{bmatrix} = \begin{bmatrix} R_1 & R_{12} \\ R_{21} & R_2 \end{bmatrix} \delta(t-s)$$

Då gäller

$$\phi_y(\omega) = H(z) \Lambda H^*(z^{-1})$$

i) $H(\infty) = I$

ii) $H(z)$ asymp. stabil

iii) $H^{-1}(z)$ (asymp.) stabil

Bestäm H och Λ ?

$$H(z) = I + C(zI - A)^{-1}K$$

$$\Lambda = CPC^T + R_2$$

$$K = (APC^T + R_{12})(CPC^T + R_2)^{-1}$$

$$P = APA^T + R_1 -$$

$$- (APC^T + R_{12})(CPC^T + R_2)^{-1}$$
$$(CPA^T + R_{12})$$

Bevisas senare

Betyder att

$$y(t) = H(q)\,\varepsilon(t) \qquad E\,\varepsilon\varepsilon^T = \Lambda\,\delta(t-s)$$

$$\begin{cases} x_\varepsilon(t+1) = A\,x_\varepsilon(t) + K\,\varepsilon(t) \\ y(t) = C\,x_\varepsilon(t) + \varepsilon(t) \end{cases}$$

Innovationsrepresentation

$$y(t) = \sum_{k=-\infty}^{t} h(t-k)\, e(k)$$

$$e(t) = \sum_{\ell=-\infty}^{t} g(t-\ell)\, y(\ell)$$

$$y(t+1) = \sum_{k=-\infty}^{t+1} h(t+1-k)\, e(k)$$

$$= \underbrace{\sum_{k=-\infty}^{t} h(t+1-k)\cdot\sum_{\ell=-\infty}^{k} g(k-\ell)\, y(\ell)}_{\text{Bästa medelkvadrat prediktion av } y(t+1)\,|\,\mathcal{Y}_t} + h(0)\overbrace{e(t+1)}^{\text{Innovation}}$$

Sammanfattning

Stationära processer

$$u \longrightarrow \boxed{S} \longrightarrow y$$

	Tidsdiskr	Tidskont
S	$H(q)$	$G(s)$
ϕ_y	$H(e^{-i\omega})H(e^{i\omega})\phi_u(\omega)$	$G(i\omega)G(-i\omega)\phi_u(\omega)$
ϕ_{uy}	$H(e^{-i\omega})\phi_u(\omega)$	$G(-i\omega)\phi_u(\omega)$
m_y	$H(1)\,m_u$	$G(0)\,m_u$
$\mathrm{Var}(u)$	$r_u(0) = \int_{-\pi}^{\pi} \phi(\omega)\,d\omega$	$r_u(0) = \int_{-\infty}^{\infty} \phi_u(\omega)\,d\omega$