6.3.1. $y(t) - 1.5y(t-1) + 0.5y(t-2) = 2\left(e(t) - 1.2e(t-1) + 0.6e(t-2)\right)$

$$y(t) = 2\,\frac{1-1.2q^{-1}+0.6q^{-2}}{1-1.5q^{-1}+0.5q^{-2}}\,e(t) = \lambda\,\frac{C^*}{A^*}\,e(t)$$

$$\hat{y}(t+k|t) = \frac{G^*}{C^*}\,y(t)$$

$$C^* = A^* F^* + q^{-k} G^*$$

$$1-1.2q^{-1}+0.6q^{-2} = (1-1.5q^{-1}+0.5q^{-2})(1+f_1q^{-1}+\ldots+f_{k-1}q^{-k+1}) +$$
$$+ q^{-k}(g_0+g_1q^{-1})$$

$k=1$: $\quad 1-1.2q^{-1}+0.6q^{-2} = (1-1.5q^{-1}+0.5q^{-2}) + q^{-1}(g_0+g_1q^{-1})$

$-1.2 = -1.5 + g_0 \Rightarrow g_0 = 0.3$

$0.6 = 0.5 + g_1 \Rightarrow g_1 = 0.1$

$\underline{\hat{y}(t+1|t) = 1.2\hat{y}(t|t-1) - 0.6\hat{y}(t-1|t-2) + 0.3y(t) \quad 0.1y(t-1)}$

$k=2$: $\quad 1-1.2q^{-1}+0.6q^{-2} = (1-1.5q^{-1}+0.5q^{-2})(1+f_1q^{-1}) + q^{-2}(g_0+g_1q^{-1})$

$-1.2 = -1.5 + f_1 \Rightarrow f_1 = 0.3$

$0.6 = 0.5 - 1.5f_1 + g_0 \Rightarrow g_0 = 0.55$

$0 = 0.5f_1 + g_1 \Rightarrow g_1 = -0.15$

$$\underline{\hat{y}(t+2|t) = \frac{0.55-0.15q^{-1}}{1-1.2q^{-1}+0.6q^{-2}}\,y(t)}$$

Allmänt: $\hat{y}(t+k|t) = \dfrac{f_k - 0.5f_{k-1}q^{-1}}{1-1.2q^{-1}+0.6q^{-2}}\,y(t) \qquad k \geq 2$

$f_k = 1.5f_{k-1} - 0.5f_{k-2} \qquad f_1 = 0.30$

$\qquad f_2 = 0.55$

5.3.2. $y(t) + a y(t-1) = \lambda\left(e(t) + c e(t-1)\right)$

$$\underbrace{(1 + a q^{-1})}_{A^*} y = \lambda \underbrace{(1 + c q^{-1})}_{C^*} e$$

$$1 + c q^{-1} = (1 + a q^{-1})(1 + f_1 q^{-1} + \ldots + f_{k-1} q^{-k+1}) + q^{-1} g_0$$

$\underline{k=1}$: $c = a + g_0 \Rightarrow g_0 = c - a$

$$\underline{\hat{y}(t+1|t) = \frac{c-a}{1+cq^{-1}} y(t)}$$

$\underline{k>1}$:

$$\begin{cases} c = a + f_1 \\ 0 = f_2 + a f_1 \\ 0 = f_3 + a f_2 \\ \vdots \\ 0 = a f_{k-1} + g_0 \end{cases} \Rightarrow \begin{cases} f_1 = c - a \\ f_2 = -a f_1 \\ \vdots \\ g_0 = -a f_{k-1} \end{cases}$$

$$\underline{\hat{y}(t+k|t)} = \frac{g_0}{1+cq^{-1}} y = \frac{-a f_{k-1}}{1+cq^{-1}} y = \underline{\frac{(-a)^{k-1}(c-a)}{1+cq^{-1}} y(t)}$$

6.3.3. $y(t) = \frac{1}{1+aq^{-1}} e(t) + \sigma v(t) = \frac{e(t) + (1+aq^{-1})\sigma v(t)}{1+aq^{-1}}$

$$e(t) + (1+aq^{-1})\sigma v(t) = \lambda(1+cq^{-1})w(t)$$

$$\begin{cases} \phi_e + \sigma^2(1+aq^{-1})(1+aq)\phi_v = \lambda^2(1+cq)(1+cq^{-1})\phi_w \\ \phi_e = \phi_v = \phi_w = 1 \end{cases}$$

$$1 + \sigma^2(1+aq+aq^{-1}+a^2) = \lambda^2(1+cq+cq^{-1}+c^2)$$

$$\begin{cases} 1+\sigma^2(1+a^2) = \lambda^2(1+c^2) \\ \sigma^2 a = \lambda^2 c \end{cases} \quad \Rightarrow \underline{\text{ger } c \text{ och } \lambda}$$

$$y(t) = \frac{\lambda(1+cq^{-1})}{1+aq^{-1}} w(t)$$

$$1+cq^{-1} = (1+aq^{-1})(1+f_1 q^{-1}) + q^{-2} g_0$$

$$c = a + f_1 \quad \Rightarrow f_1 = c-a$$

$$0 = af_1 + g_0 \Rightarrow g_0 = -a(c-a)$$

$$\hat{y}(t+2|t) = \frac{-a(c-a)}{1+cq^{-1}} y(t)$$

6.3.4. $y(t) + 0.7\,y(t-1) = e(t) + 2e(t-1)$

$$y(t) = \frac{1+2q^{-1}}{1+0.7q^{-1}}\, e(t)$$

$$\begin{cases} (1+2q^{-1})e(t) = \lambda(1+cq^{-1})\,w(t) & |c|<1 \\ \phi_e = \phi_w = 1 \end{cases}$$

$$(1+2q^{-1})(1+2q) = \lambda^2(1+cq^{-1})(1+cq)$$

$$1 + 2q^{-1} + 2q + 4 = \lambda^2(1 + cq^{-1} + cq + c^2)$$

$$\begin{cases} 5 = \lambda^2(1+c^2) \\ 2 = \lambda^2 c \qquad \Rightarrow \lambda^2 = \frac{2}{c} \end{cases}$$

$$5 = \frac{2}{c}(1+c^2) = \frac{2}{c} + 2c$$

$$c^2 - \frac{5}{2}c + 1 = 0 \quad \Rightarrow c = \frac{5}{4}\underset{(+)}{-} \pm \sqrt{\frac{25}{16} - 1} = \frac{5}{4} - \frac{3}{4} = \frac{1}{2}$$

$$\begin{cases} c = \frac{1}{2} \\ \lambda = 2 \end{cases}$$

$$y(t) = \frac{2(1+\frac{1}{2}q^{-1})}{1+0.7q^{-1}}\, e(t)$$

$$1 + 0.5q^{-1} = 1 + 0.7q^{-1} + q^{-1}g_0$$

$$0.5 = 0.7 + g_0 \quad \Rightarrow g_0 = -0.2$$

$$\underline{\hat{y}(t+1|t) = -0.5\,\hat{y}(t|t-1) - 0.2\,y(t)}$$

$$\mathrm{Var}\,\tilde{y}(t+1|t) = \lambda^2 = \underline{\underline{4}}$$

6.3.5. $$y(t) = \sum_{k=t_0}^{t} g(t,k)e(k) \qquad e(t) = \sum_{k=t_0}^{t} h(t,k)y(k)$$

$$y(t+k) = \sum_{\ell=t_0}^{t+k} g(t+k,\ell)e(\ell) = \sum_{\ell=t_0}^{t} g(t+k,\ell)e(\ell) + \sum_{\ell=t+1}^{t+k} g(t+k,\ell)e(\ell)$$

$$= \sum_{\ell=t_0}^{t} g(t+k,\ell)\sum_{n=t_0}^{\ell} h(\ell,n)y(n) + \sum_{\ell=t+1}^{t+k} g(t+k,\ell)e(\ell) =$$

$$= \underbrace{\sum_{\ell=t_0}^{t}\sum_{n=t_0}^{\ell} g(t+k,\ell)h(\ell,n)y(n)}_{\underline{\hat{y}(t+k\mid t)}} + \sum_{\ell=t+1}^{t+k} g(t+k,\ell)e(\ell)$$

6.3.6. $$y(t) = \lambda \frac{C^*}{A^*} e(t)$$

$$\begin{cases} A(z) = a_0 z^n + a_1 z^{n-1} + \dots + a_n \\ C(z) = c_0 z^m + c_1 z^{m-1} + \dots + c_m \end{cases}$$

$$\hat{y}(t+k\mid t) = \frac{G^*}{C^*} y$$

G^* har ordn. $n-1$

C^* har ordn. m

$\Rightarrow$ Systemet har ordningen $\max(m, n-1)$

6.3.7. $(1-2.6q^{-1}+2.85q^{-2}-1.4q^{-3}+0.25q^{-4})\,y(t) = (1-0.7q^{-1})\,e(t)$

$$1-0.7q^{-1} = 1-2.6q^{-1}+2.85q^{-2}-1.4q^{-3}+0.25q^{-4}+q^{-1}(g_0+g_1q^{-1}+g_2q^{-2}+g_3q^{-3})$$

$$\begin{cases} -0.7 = -2.6+g_0 \\ 0 = 2.85+g_1 \\ 0 = -1.4+g_2 \\ 0 = 0.25+g_3 \end{cases} \Rightarrow \begin{cases} g_0 = 1.9 \\ g_1 = -2.85 \\ g_2 = 1.4 \\ g_3 = -0.25 \end{cases}$$

$$\hat{y}(t+1|t) = \frac{1.9-2.85q^{-1}+1.4q^{-2}-0.25q^{-3}}{1-0.7q^{-1}}$$

Dynamiken i prediktorn är av fjärde ordningen

6.4.1. $y(t) = \dfrac{1}{1+0.5q^{-1}} u(t-1) + \dfrac{1+0.7q^{-1}}{1-0.2q^{-1}}$

$$(1+0.3q^{-1}-0.1q^{-2})\,y(t) = (1-0.2q^{-1})\,u(t-1) + (1+1.2q^{-1}+0.35q^{-2})$$

$$\begin{cases} A^*(q^{-1}) = 1+0.3q^{-1}-0.1q^{-2} \\ B^*(q^{-1}) = 1-0.2q^{-1} \\ C^*(q^{-1}) = 1+1.2q^{-1}+0.35q^{-2} \end{cases}$$

$$(1+1.2q^{-1}+0.35q^{-2}) = (1+0.3q^{-1}-0.1q^{-2}) + q^{-1}(g_0+g_1q^{-1})$$

$1.2 = 0.3 + g_0 \quad \Rightarrow g_0 = 0.9$

$0.35 = -0.1 + g_1 \quad \Rightarrow g_1 = 0.45$

$$u(t) = -\frac{G^*}{B^*F^*}\,y(t) = -\frac{0.9+0.45q^{-1}}{1-0.2q^{-1}}\,y(t)$$

Anm. C skall ha rötter innanför enhetscirkeln.

$z^2 + 1.2z + 0.35 = 0$

$z = -0.6 \pm \sqrt{0.36-0.35} = 0.6 \pm 0.1 \quad$ O.K.

6.4.2. $y(t)+ay(t-1)=bu(t-k)+\lambda(e(t)+ce(t-1))$

$$(1+aq^{-1})y(t)=bu(t-k)+\lambda(1+cq^{-1})e(t)$$

$\underline{k=1}$: $1+cq^{-1}=1+aq^{-1}+q^{-1}g_0$

$$g_0=c-a$$

$$u(t)=-\frac{g_0}{b}=\frac{a-c}{b}y(t)$$

$$\underline{\operatorname{Var}(y)=\lambda^2}$$

$\underline{k=2}$: $1+cq^{-1}=(1+aq^{-1})(1+f_1q^{-1})+q^{-2}g_0$

$$c=a+f_1 \Rightarrow f_1=c-a$$

$$0=af_1+g_0 \Rightarrow g_0=a(a-c)$$

$$u(t)=-\frac{g_0}{bF^*}=\frac{a(c-a)}{b(1+(c-a)q^{-1})}y(t)$$

$$\underline{\operatorname{Var}(y)=\lambda^2(1+(c-a)^2)}$$

$\underline{k=3}$: $1+cq^{-1}=(1+aq^{-1})(1+f_1q^{-1}+f_2q^{-2})+q^{-3}g_0$

$$c=a+f_1 \Rightarrow f_1=c-a$$

$$0=af_1+f_2 \Rightarrow f_2=a(a-c)$$

$$0=af_2+g_0 \Rightarrow g_0=a^2(c-a)$$

$$u(t)=-\frac{g_0}{bF^*}=\frac{a^2(a-c)}{b(1+(c-a)q^{-1}+a^2(c-a)q^{-2})}y(t)$$

$$\underline{\operatorname{Var}(y)=\lambda^2(1+(c-a)^2+a^2(a-c)^2)}$$

6.4.3. $A^* y(t) = B^* u(t-k)$ $\qquad \underline{t > k}$

$$u(t) = -\frac{G^*}{B^* F^*} y(t)$$

$$1 = A^* F^* + q^{-k} G^*$$

$$A^* y(t) = B^* q^{-k} u(t) = -\frac{q^{-k} B^* G^*}{B^* F^*} y(t) = -\frac{1 - A^* F^*}{F^*} y(t)$$

$$y(t) = 0$$

$\Rightarrow y(t) = 0$ efter k steg #

6.4.4. $A^* y(t) = B^* u(t-k) + \lambda C^* e(t)$

$$u(t) = -\frac{G^*}{B^* F^*} y(t)$$

$$1 = A^* F^* + q^{-k} G^*$$

$$A^* y(t) = -\frac{q^{-k} B^* G^*}{B^* F^*} y(t) + \lambda C^* e(t) = \frac{A^* F^* - 1}{F^*} y(t) + \lambda C^* e(t)$$

$$A^* F^* y(t) - A^* F^* y(t) + y(t) = \lambda C^* F^* e(t)$$

$$y(t) = \lambda C^* F^* e(t)$$

$$\underline{E y^2(t) = \lambda^2 (1 + (c_1 + f_1)^2 + (c_2 + f_2 + c_1 f_1)^2 + \dots)}$$

6.4.7. $y(t) + 0.5\,y(t-1) = u(t-1) + e(t) + 2e(t-1)$

$$(1+0.5q^{-1})y(t) = u(t-1) + (1+2q^{-1})e(t)$$

C^* har nollställe utanför enhetscirkeln.

$$\begin{cases} (1+2q^{-1})e(t) = \lambda(1+cq^{-1})w(t) \\ \Phi_e = \Phi_w = 1 \end{cases}$$

$$(1+2q^{-1})(1+2q) = \lambda^2(1+cq^{-1})(1+cq)$$

$$\begin{cases} \lambda^2(1+c^2) = 5 \\ \lambda^2 c = 2 \\ |c| < 1 \end{cases} \Rightarrow \begin{cases} \lambda = 2 \\ c = \frac{1}{2} \end{cases}$$

$$(1+0.5q^{-1})y(t) = u(t-1) + 2(1+0.5q^{-1})w(t)$$

$$1+0.5q^{-1} = (1+0.5q^{-1}) + q^{-1}g_0$$

$$g_0 = 0$$

$\Rightarrow$ Minimalvariansstyrlagen : $\underline{u(t) = 0}$

$$E\,y^2 = \lambda^2 \cdot 1 = \underline{4}$$

6.4.8. $y(t) = \dfrac{1}{1+2q^{-1}} u(t-1) + \dfrac{1+0.7q^{-1}}{1-0.2q^{-1}} e(t)$

$$\underbrace{(1+1.8q^{-1}-0.4q^{-2})}_{A^*} y(t) = \underbrace{(1-0.2q^{-1})}_{B^*} u(t-1) + \underbrace{(1+2.7q^{-1}+1.4q^{-2})}_{C^{*\prime}} e(t)$$

C' rötter innanför enhetscirkeln?: $z^2 + 2.7z + 1.4 = 0$

$z = -1.35 \pm \sqrt{1.35^2 - 1.4}$ <u>nej!</u>

$$(1+0.7q^{-1})(1+2q^{-1})e(t) = \lambda(1+0.7q^{-1})(1+cq^{-1})w(t)$$

Enl. 6.4.7: $\begin{cases} \lambda = 2 \\ c = 0.5 \end{cases}$

$$C^* = (1+0.7q^{-1})(1+0.5q^{-1}) = 1 + 1.2q^{-1} + 0.35q^{-2}$$

$$C^* = A^*F^* + q^{-k}G^*$$

$$1 + 1.2q^{-1} + 0.35q^{-2} = (1+1.8q^{-1}-0.4q^{-2}) + q^{-1}(g_0 + g_1 q^{-1})$$

$1.2 = 1.8 + g_0 \quad \Rightarrow g_0 = -0.6$

$0.35 = -0.4 + g_1 \quad \Rightarrow g_1 = 0.75$

$$u(t) = -\frac{G^*}{B^*F^*} y(t) = \underline{\frac{0.6 - 0.75q^{-1}}{1-0.2q^{-1}} y(t)}$$

$E y^2 = \lambda^2 \cdot 1 = \underline{\underline{4}}$

6.4.9. $u(t) = u(t-1) + K\left[\left(1+\frac{h}{T}\right)y(t) - y(t-1)\right]$

$$u(t) = \frac{K(1+\frac{h}{T}) - Kq^{-1}}{1+q^{-1}}\,y(t) = -\frac{G^*}{B^*F^*}\,y(t) = -\frac{G^*}{B^*}\,y(t)$$

(I) $B^* = 1+q^{-1} \Rightarrow F^* = 1 \Rightarrow k = 1$

$C^* = A^*F^* + q^{-1}G^*$

$(1 + c_1 q^{-1} + c_2 q^{-2}) = (1 + a_1 q^{-1} + a_2 q^{-2}) + q^{-1}\left(-K(1+\frac{h}{T}) + Kq^{-1}\right)$

Ger c_1, c_2, a_1 och a_2.

$A^* y(t) = B^* u(t-1) + \lambda C^* e(t)$

(II) $B^* = 1 \Rightarrow F^* = 1+q^{-1} \Rightarrow k = 2$

$C^* = A^*F^* + q^{-2}G^*$

$(1 + c_1 q^{-1} + c_2 q^{-2}) = (1 + a_1 q^{-1} + a_2 q^{-2})(1+q^{-1}) + q^{-2}\left(-K(1+\frac{h}{T}) + Kq^{-1}\right)$

Ger c_1, c_2, a_1 och a_2.

$A^* y(t) = B^* u(t-1) + \lambda C^* e(t)$

6.4.12. $A^* y(t) = B^* u(t-k) + \lambda C^* e(t)$

$$y(t) = \frac{B^*}{A^*} u(t-k) + \lambda \frac{C^*}{A^*} e(t)$$

$$C^* = A^* F_1^* + q^{-(k+1)} G_1^*$$

F_1^* har grad k

G_1^* har grad $n-1$

$$y(t) = \frac{B^*}{A^*} u(t-k) + \lambda F_1^* e(t) + \lambda q^{-(k+1)} \frac{G_1^*}{A^*} e(t)$$

$$\begin{cases} y(t+k) = \dfrac{B^*}{A^*} u(t) + \lambda F_1^* e(t+k) + \lambda q^{-1} \dfrac{G_1^*}{A^*} e(t) \\ \lambda e(t) = \dfrac{A^*}{C^*} y(t) - \dfrac{B^*}{C^*} q^{-k} u(t) \end{cases}$$

$$y(t+k) = \frac{B^*}{A^*} u(t) + \lambda F_1^* e(t+k) + q^{-1} \frac{G_1^*}{C^*} y(t) - q^{-(k+1)} \frac{B^* G_1^*}{A^* C^*} u(t) =$$

$$= \lambda F_1^* e(t+k) + q^{-1} \frac{G_1^*}{C^*} y(t) + \frac{B^* F_1^*}{C^*} u(t)$$

Välj $\underline{u(t) = -q^{-1} \dfrac{G_1^*}{B^* F_1^*} y(t)}$

6.5.1. $\underbrace{(1+0.64q^{-1}+0.22q^{-2})}_{A^*}y(t) = \underbrace{(6.4+19.2q^{-1})}_{B^*}u(t-3)+\lambda\underbrace{(1-0.82q^{-1}+0.21q^{-2})}_{C^*}e(t)$

Minimalvariansreglering

$$1-0.82q^{-1}+0.21q^{-2} = (1+0.64q^{-1}+0.22q^{-2})(1+f_1q^{-1}+f_2q^{-2})+q^{-3}(g_0+g_1q^{-1})$$

$$\begin{cases} -0.82 = 0.64+f_1 \\ 0.21 = 0.22+f_2+0.64f_1 \\ 0 = f_2\cdot 0.64+0.22f_1+g_0 \\ 0 = 0.22f_2+g_1 \end{cases} \Rightarrow \begin{cases} f_1 = -1.46 \\ f_2 = 0.9244 \\ g_0 = -0.270416 \\ g_1 = -0.203368 \end{cases}$$

$$u(t) = -\frac{g_0+g_1q^{-1}}{B^*F^*} = -\frac{-0.270416-0.203368q^{-1}}{6.4(1+3q^{-1})(1-1.46q^{-1}+0.9244q^{-2})}$$

$$\text{Var}\, y = \lambda^2(1+1.46^2+0.9244^2) \approx \underline{3.99\lambda^2}$$

B har ett nollställe utanför enhetscirkeln ⇒

⇒ Icke min. fas ⇒ Känsligt för parametervariationer.

Suboptimal reglering

$$B^* = B_1^*B_2^* = 6.4(1+3q^{-1})$$

$$C^* = A^*F'^* + q^{-k}B_2^*G'^*$$

$$1-0.82q^{-1}+0.21q^{-2} = (1+0.64q^{-1}+0.22q^{-2})(1+f_1q^{-1}+f_2q^{-2}+f_3q^{-3}) +$$
$$+q^{-3}(1+3q^{-1})(g_0+g_1q^{-1})$$

$$\begin{cases} -0.82 = 0.64 + f_1 & \Rightarrow f_1 = -1.46 \\ 0.21 = 0.64 f_1 + 0.22 + f_2 & \Rightarrow f_2 = 0.9244 \\ 0 = 0.64 f_2 + 0.22 f_1 + f_3 + g_0 & \\ 0 = 0.64 f_3 + 0.22 f_2 + g_1 + 3 g_0 & \\ 0 = 0.22 f_3 + 3 g_1 & \end{cases}$$

$$f_3 + g_0 = -0.270416$$

$$\left. \begin{array}{l} 0.64 f_3 + 3g_0 + g_1 = 0.203368 \\ g_1 = -0.66 f_3 \end{array} \right\} \Rightarrow -0.02 f_3 + 3 g_0 = 0.203368$$

$$-0.02 f_3 + 3(-0.270416 - f_3) = 0.203368$$

$$f_3 \approx -0.33596556$$

$$g_0 \approx 0.06554956$$

$$g_1 \approx 0.22173727$$

$$u(t) = -\frac{G'^*}{B_1^* F_1^*} y(t) = -\frac{0.06555 + 0.22174 q^{-1}}{6.4(-1.46 + 0.9244 q^{-1} - 0.33597 q^{-2})} y(t)$$

$$\underline{\text{Var}\, y} = (1 + 1.46^2 + 0.9244^2 + 0.33596556^2)\lambda^2 \approx \underline{4.09 \lambda^2}$$

6.5.2. 5.11 ger $y_1(t) = \lambda[e(t) + f_1 e(t-1) + \dots + f_{k-1} e(t-k+1) +$

$+ f_k' e(t-k) + \dots + f_{k+n_2-1}' e(t-k-n_2+1)]$

där $C^* = A^* F'^* + q^{-k} B_2^* G'^*$

3.25 ger $y_2(t+k+n_2-1|t) = \lambda[e(t+k+n_2-1) + f_1 e(t+k+n_2-2) + \dots +$

$+ f_{k+n_2-2} e(t-1)]$

där $C^* = A^* F^* + q^{-k-n_2+1} G^*$

$\text{Var } y_1(t) = \lambda^2 [1 + f_1'^2 + f_2'^2 + \dots + f_{k+n_2-1}'^2]$

$\text{Var } y_2(t) = \lambda^2 [1 + f_1^2 + f_2^2 + \dots + f_{k+n_2-2}^2]$

6.5.3. $H(q) C(q) = A(q) F'(q) + B_2(q) G'(q)$

$A y(t) = B u(t-k) + \lambda C e(t)$

$u(t) = -\frac{q^k G'}{B_1 F'} y(t)$

$A y(t) = -\frac{B G'}{B_1 F'} y(t) + \lambda C e(t)$

$\left[A + \frac{B G'}{B_1 F'}\right] y(t) = \lambda C e(t)$

$\left[A + \frac{B_2 G'}{F'}\right] y(t) = \lambda C e(t)$

$[A F' + B_2 G'] y(t) = \lambda C F' e(t)$

$\underline{H y(t) = \lambda F' e(t)}$

$$\left[A + \frac{B G'}{B_1 F'}\right] y(t) = \lambda C e(t)$$

$$\left[A B_1 F_1 + B_1 B_2 G'\right] y(t) = \lambda C B_1 F' e(t)$$

$$B_1 \left[A F_1 + B_2 G'\right] y(t) = \lambda C B_1 F' e(t)$$

$$B_1 H C \; y(t) = \lambda C B_1 F' e(t)$$

$\Rightarrow$ Karakteristiska polynomet = $\underline{B_1 H C}$

5.5.4. $$A^* y(t) = B^* u(t-k) + \lambda C^* e(t)$$

$$u(t) = -\frac{G_1^*}{F_1^*} y(t)$$

$$C^* = A^* F_1^* + q^{-k} B^* G_1^*$$

$$A^* y(t) = -\frac{q^{-k} B^* G_1^*}{F_1^*} + \lambda C^* e(t)$$

$$\left[A^* F_1^* + q^{-k} B^* G_1^*\right] y(t) = \lambda C^* e(t)$$

$$C^* y(t) = \lambda C^* e(t)$$

$$q^{n+k-1} C = A F_1 + B G_1$$

$$q^{n+k-1} C y(t) = \lambda q^{n+k-1} e(t)$$

$\Rightarrow$ Kar. pol = $\underline{q^{n+k-1} C}$

$\underline{\text{Var } y = \lambda^2 (1 + f_1'^2 + \ldots + f_{k+n-1}'^2)}$

$\underline{\text{Min var } y = \lambda^2 (1 + f_1'^2 + \ldots + f_{k-1}'^2)}$

6.5.5. $(1+aq^{-1})y(t) = (1+2.5q^{-1}+q^{-2})u(t-1) + (1+cq^{-1})e(t)$

Minimalvariansstrategi

$1+cq^{-1} = (1+aq^{-1}) + q^{-1}g_0$

$c = a+g_0 \Rightarrow g_0 = c-a$

$$u(t) = \frac{a-c}{(1+2.5q^{-1}+q^{-2})}y(t)$$

reglerfelet $y(t) = e(t)$

$z^2+2.5z+1 = 0$

$z = -1.25 \pm\sqrt{1.25^2-1} = -1.25 \pm 0.75 = -2, -0.5$

B har nollställe utanför enhetscirkeln $\Rightarrow$ icke min. fas $\Rightarrow$
$\Rightarrow$ känsligt för parametervariationer.

Suboptimal strategi 1

$B^* = B_1^* B_2^* = (1+0.5q^{-1})(1+2q^{-1})$

$C^* = AF'^* + q^{-1}B_2^* G'^*$

$1+cq^{-1} = (1+aq^{-1})(1+f_1q^{-1}) + q^{-1}(1+2q^{-1})g_0$

$$\begin{cases} c = a+f_1+g_0 \\ 0 = af_1+2g_0 \end{cases} \Rightarrow \begin{cases} f_1 = \dfrac{2(c-a)}{2-a} \\ g_0 = -\dfrac{a(c-a)}{2-a} \end{cases}$$

$$u(t) = \frac{-\dfrac{a(c-a)}{2-a}}{\left(1+\dfrac{2(c-a)}{2-a}q^{-1}\right)(1+0.5q^{-1})}y(t)$$

reglerfelet $y(t) = e(t) + \dfrac{2(c-a)}{2-a}e(t-1)$

Suboptimal strategi 2

$$C^* = A^* F_1^* + q^{-1} B^* G_1^*$$

$$1 + cq^{-1} = (1 + aq^{-1})(1 + f_1 q^{-1} + f_2 q^{-2}) + q^{-1}(1 + 2.5q^{-1} + q^{-2})\, g_0$$

$$\begin{cases} c = a + f_1 + g_0 \\ 0 = a f_1 + f_2 + 2.5 g_0 \\ 0 = a f_2 + g_0 \end{cases} \Rightarrow \begin{cases} f_1 + g_0 = c - a \\ a f_1 - g_0/a + 2.5 g_0 = 0 \\ f_2 = -g_0/a \end{cases}$$

$$a(c - a - g_0) - g_0/a + 2.5 g_0 = 0$$

$$(-a - \tfrac{1}{a} + 2.5)\, g_0 = -a(c-a)$$

$$\underline{g_0 = \frac{a^2(c-a)}{a^2 - 2.5a + 1}}$$

$$\underline{f_2 = -\frac{a(c-a)}{a^2 - 2.5a + 1}}$$

$$f_1 = c - a - g_0 = \frac{(c-a)(a^2 - 2.5a + 1) - a^2(c-a)}{a^2 - 2.5a + 1} = \underline{\frac{(c-a)(1 - 2.5a)}{a^2 - 2.5a + 1}}$$

$$\underline{u(t) = -\frac{G_1^*}{F_1^*}\, y(t) = -\frac{\dfrac{a^2(c-a)}{a^2 - 2.5a + 1}}{1 + \dfrac{(c-a)(1-2.5a)}{a^2 - 2.5a + 1} q^{-1} - \dfrac{a(c-a)}{a^2 - 2.5a + 1} q^{-2}}\, y(t)}$$

$$\underline{\text{Reglerfelet} = y(t) = e(t) + \frac{(c-a)(1-2.5a)}{a^2 - 2.5a + 1} e(t-1) - \frac{a(c-a)}{a^2 - 2.5a + 1} e(t-2)}$$