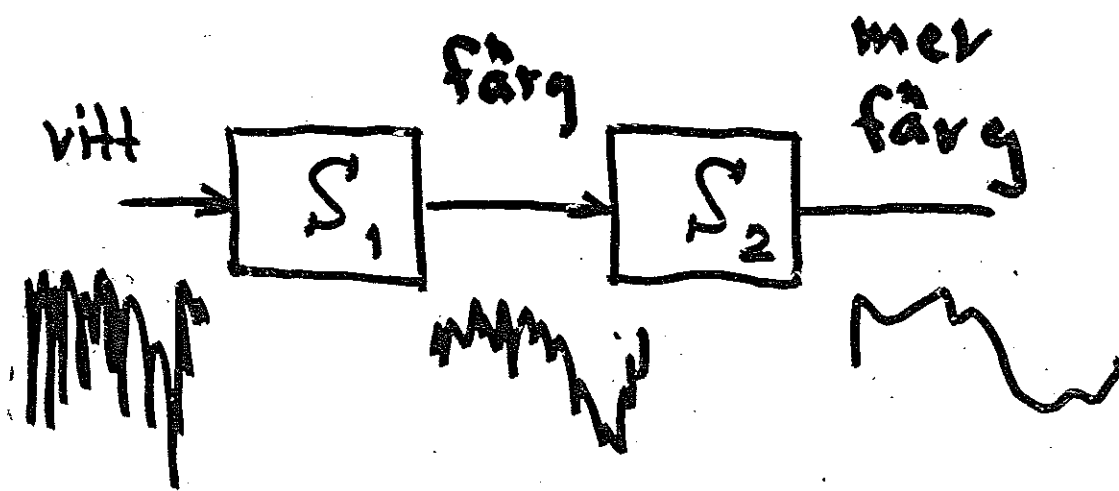


Dagens idé:



Bara en bruskälla

11.

SPEKTRALTÄTHET

KJA s 27

$$F(\omega) = F_a(\omega) + F_d(\omega) + F_s(\omega)$$

$$\int_{-\infty}^{\omega} \phi(\omega') d\omega'$$

step fkn

$$\sum_{\omega_y \leq \omega} F(\omega_y)$$

— singular part
konst nästan
överallt

$\phi(\omega)$ Spektral(täthet)

$$\phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} r(t) dt$$

$$r(t) = \int_{-\infty}^{\infty} e^{i\omega t} \phi(\omega) d\omega$$

$$\phi(\omega) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} r(n) e^{-in\omega}$$

$$r(n) = \int_{-\pi}^{\pi} e^{in\omega} \phi(\omega) d\omega$$

Exempel 1

$$\phi(\omega) = \frac{1.04 + 0.4 \cos \omega}{1.25 + \cos \omega} = \frac{(e^{i\omega} + 0.2)(e^{-i\omega} + 0.2)}{(e^{i\omega} + 0.5)(e^{-i\omega} + 0.5)}$$

$$\phi(z) = \frac{(z + 0.2)(z^{-1} + 0.2)}{(z + 0.5)(z^{-1} + 0.5)} = H_1(z) H_1(z^{-1})$$

$$= \frac{1 + 0.2z}{z + 0.5} \cdot \frac{1 + 0.2z^{-1}}{z^{-1} + 0.5} = H_2(z) H_2(z^{-1})$$

$$= \frac{z + 0.2}{1 + 0.5z} \cdot \frac{z^{-1} + 0.2}{1 + 0.5z^{-1}} = H_3(z) H_3(z^{-1})$$

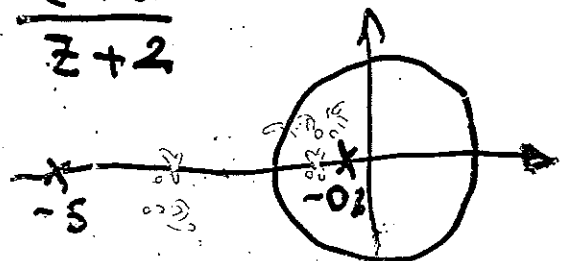
$$= \frac{1 + 0.2z}{1 + 0.5z} \cdot \frac{1 + 0.2z^{-1}}{1 + 0.5z^{-1}} = H_4(z) H_4(z^{-1})$$

$$H_1 = \frac{z + 0.2}{z + 0.5}$$

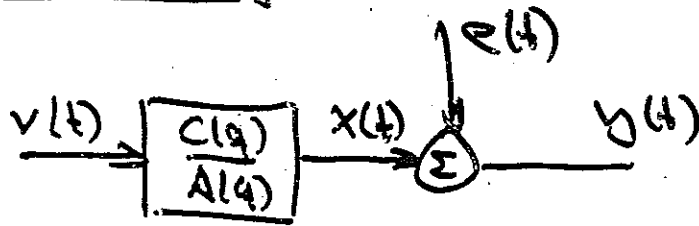
$$H_2 = \frac{1 + 0.2z}{z + 0.5} = 0.2 \frac{z + 5}{z + 0.5}$$

$$H_3 = \frac{z + 0.2}{1 + 0.5z} = 2 \frac{z + 0.2}{z + 2}$$

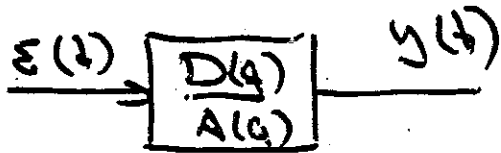
$$H_4 = \frac{1 + 0.2z}{1 + 0.5z} = 0.4 \frac{z + 5}{z + 2}$$



Tolkning



e, v obero



$$\phi_y(z) = \lambda_e^2 + \lambda_v^2 \frac{C C^*}{A A^*} = \lambda_\varepsilon^2 \frac{D D^*}{A A^*}$$

$$\lambda_\varepsilon^2 D D^* = \lambda_v^2 C C^* + \lambda_e A A^*$$

$n+1$ ekv för $n+1$ obero.

Räcker med en brus-källa ε ∇

Spektralfaktorerering - Tillståndsmodell

$$x(t+1) = Ax(t) + v(t)$$

$$|\lambda(A)| < 1$$

$$y(t) = Cx(t) + e(t)$$

$$E \begin{bmatrix} v \\ e \end{bmatrix} \begin{bmatrix} v^T & e^T \end{bmatrix} = \begin{bmatrix} R_1 & R_{12} \\ R_{21} & R_2 \end{bmatrix} \delta(t-s)$$

Dä gäller

$$\phi_y(w) = H(z) \wedge H^*(z^{-1})$$

i) $H(\infty) = I$

ii) $H(z)$ asymp. stabil

iii) $H^{-1}(z)$ (asymp.) stabil

Bestäm H och Λ ?

$$H(z) = I + C(zI - A)^{-1}K$$

$$\Lambda = CPC^T + R_2$$

$$K = (APC^T + R_{12})(CPC^T + R_2)^{-1}$$

$$P = APA^T + R_1 -$$

$$- (APC^T + R_{12})(CPC^T + R_2)^{-1} \\ (CPA^T + R_{12})$$

Revised senate

Betyder att

$$y(t) = H(q) \varepsilon(t) \quad E \varepsilon \varepsilon^T = \Lambda \delta(t-s)$$

$$\begin{cases} x_\varepsilon(t+1) = Ax_\varepsilon(t) + K\varepsilon(t) \\ y(t) = Cx_\varepsilon(t) + \varepsilon(t) \end{cases}$$

Innovations representation

$$y(t) = \sum_{k=-\infty}^t h(t-k) e(k)$$

$$e(t) = \sum_{l=-\infty}^t g(t-l) y(l)$$

$$y(t+1) = \sum_{k=-\infty}^{t+1} h(t+1-k) e(k)$$

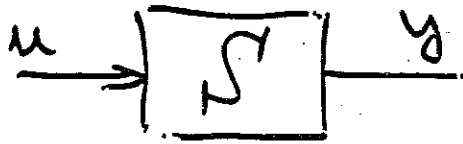
Innovation
↓

$$= \sum_{k=-\infty}^t h(t+1-k) \cdot \sum_{l=-\infty}^k g(k-l) y(l) + \underbrace{h(0)}_{(e(t+1))}$$

Bästa medelkvadrat
prediktion av $y(t+1) | \mathcal{Y}_t$

Sammanfattning

Stationära processer



	Tidsdiskr	Tidskont
Σ	$H(q)$	$G(s)$
Φ_y	$H(e^{-i\omega})H(e^{i\omega})\Phi_u(\omega)$	$G(i\omega)G(-i\omega)\Phi_u(\omega)$
Φ_{uy}	$H(e^{-i\omega})\Phi_u(\omega)$	$G(-i\omega)\Phi_u(\omega)$
m_y	$H(1)m_u$	$G(0)m_u$
$\text{Var}(u)$	$r_u(0) = \int_{-\pi}^{\pi} \phi(\omega) d\omega$	$r_u(0) = \int_{-\infty}^{\infty} \phi_u(\omega) d\omega$