(1)

FÖRELÄSNING 3

- Vad kan gå snett?
- Sampling av tidskontinuerligt system
- Sammanfattning

(2)

STOKASTISKA TILLST. MODELLER

$$x(t+1) = g(x,t) + \sigma(x,t)\,e(t)$$

$\llcorner$ $N(0,1)$

$$dx = f(x,t)\,dt + \sigma(x,t)\,dw$$

$\llcorner$ $W(I\,dt)$

$$x(t) = x(t_0) + \int_{t_0}^{t} f(x(s),s)\,ds + \int_{t_0}^{t} \sigma(x(s),s)\,dw(s)$$

STOKASTISKA INTEGRALER

$$\int f(t)\,dy(t)$$

- $f(t)$ determ. fkn
- $f(t)$ stok proc

f,y ober

f,y beroende

Ito integral

(3)

Th 6.1

$$dx = A(t)\,x\,dt + dv$$

$$\frac{dm_x}{dt} = A(t)\,m_x$$

$$m_x(t_0) = m_0$$

$$R(s,t) = \begin{cases} \phi(s,t)\,P(t) & s \geq t \\ P(s)\,\phi^T(s,t) & s \leq t \end{cases}$$

$$\frac{dP}{dt} = AP + PA^T + R$$

$$P(t_0) = R_0$$

(4)

LIVREMMEN NÖDVÄNDIG?

$$\dot{x} = Ax + e \qquad \mathrm{cov}(e(t), e(s)) = R_1\,\delta(t-s)$$

$$P(t) = E\{x(t)x^T(t)\}$$

$$\frac{dP}{dt} = E\{\frac{dx}{dt}x^T\} + E\{x\frac{dx^T}{dt}\}$$

$$= E\{Axx^T + ex^T\} + E\{xx^TA^T + xe^T\}$$

$$= AP + 0 + PA^T + 0$$

FEL $\frac{dx}{dt}$ existerar inte i medelkvadratmening

Hur gör man rätt?

Titta på differenser och gör gränsövergång ($\Delta x = Ax\Delta t + \Delta v$)

$$\Delta(xx^T) = (x + \Delta x)(x + \Delta x)^T - xx^T$$

$$= x\,\Delta x^T + \Delta x\, x^T + \underbrace{\Delta x(\Delta x)^T}_{\text{farliga termen}}$$

$$E\,\Delta(xx^T) = \Delta E\, xx^T = \Delta P$$

$$= E\{x(Ax\Delta t + \Delta v)^T\} + E\{(Ax\Delta t + \Delta v)x^T\}$$

$$+ E\{(Ax\Delta t + \Delta v)(Ax\Delta t + \Delta v)^T\}$$

$$= (E\,xx^T A^T + A E\,xx^T)\Delta t$$

$$+ E\,\Delta v \Delta v^T + o(\Delta t)$$

$\Delta t \to 0$

$$\frac{dP}{dt} = PA^T + AP + R_1$$

RÄTT!

Olinjär stochastic diff. eq s71

Fokker-Planck ekv

$$dx = f(x,t)\,dt + \sigma(x,t)\,dw$$

$$\frac{\partial p}{\partial t} = -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}(p \cdot f_i) + \frac{1}{2}\sum_{i,j,k=1}^{n} \frac{\partial^2}{\partial x_i \partial x_j}(p\sigma_{ik}\sigma_{jk}$$

$$p(x, t_0, x_0, t_0) = \delta(x - x_0)$$

Ito's differentieringsregel s74

$y(x,t)$ kont diffbar i t & 2 ggr i x

$$dy = [y_t + y_x^T f + \frac{1}{2}\mathrm{tr}(y_{xx}\sigma\sigma^T)]dt + y_x^T \sigma\, dw$$

7

Exempel

$$dx = Ax\,dt + dv$$

$$y = x^T S(t) x$$

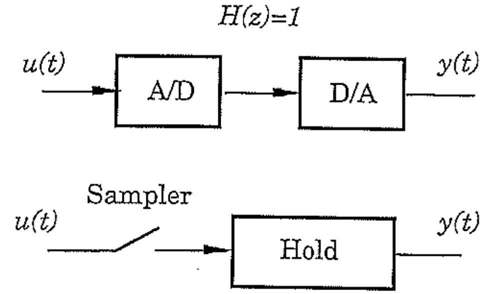
$$dy = \left[x^T \frac{dS}{dt} x + x^T A^T S x + x^T S A x + \operatorname{tr} S R_1 \right] dt + dv^T S x + x^T S\, dv$$

Även

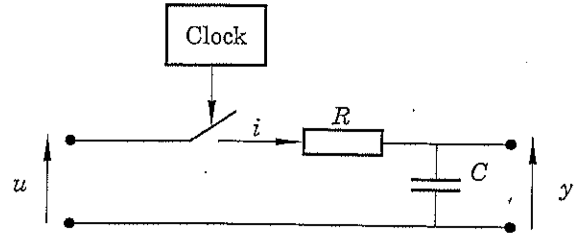
$$\int_{t_0}^{t} x(s)^T S(s) x(s)\, ds$$

8

Model of sample and hold



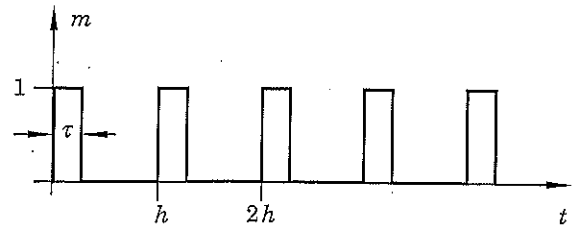
Model



$$m(t) = \begin{cases} 1 & \text{if switch is closed} \\ 0 & \text{if switch is open} \end{cases}$$

$$i = \frac{u - y}{R}\, m$$

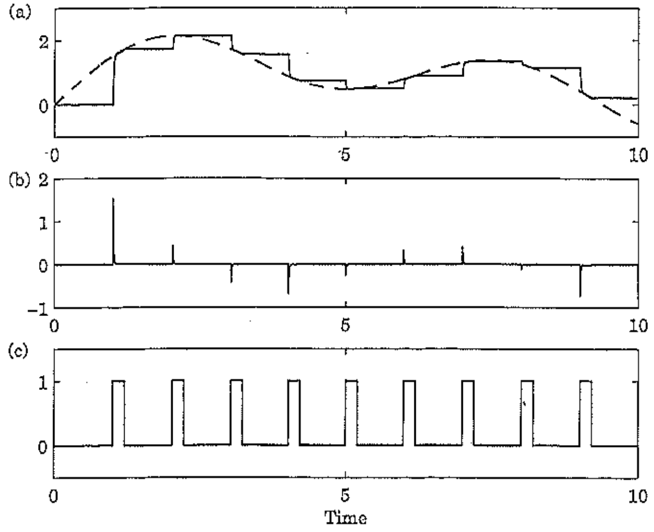
$$C\,\frac{dy(t)}{dt} = i(t) = \frac{u(t) - y(t)}{R}\, m(t)$$



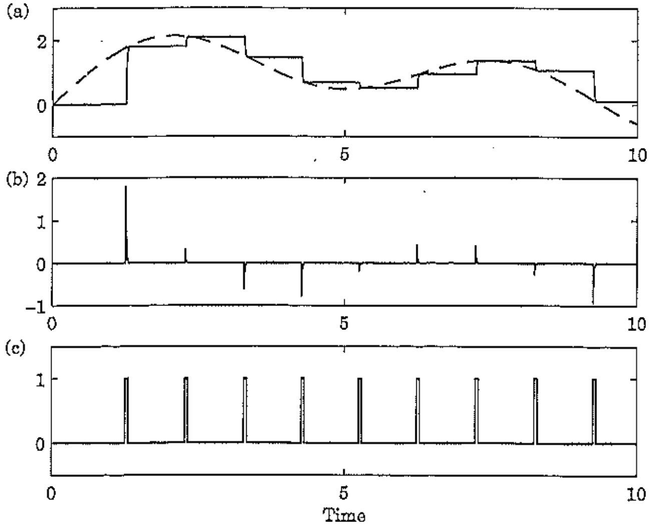
(9)

Compromise between τ and RC

$\tau = 0.2$ and $RC = 0.01$



$\tau = 0.05$ and $RC = 0.01$



(10)

Sampling av stoch. diff. ekv. 1.

Modell

$$dx = Ax\,dt + dv \qquad R_1, R_2$$
$$dy = Cx\,dt + de \qquad e, v \text{ ober}$$

Integrera över ett intervall

$$x(t_{i+1}) = \phi(t_{i+1}, t_i)\,x(t_i) + \tilde{v}(t_i)$$
$$z(t_{i+1}) = y(t_{i+1}) - y(t_i) = \Theta(t_{i+1}, t_i)\,x(t_i) + \tilde{e}(t_i)$$

ϕ fundamentalmatrisen

$$\tilde{v}(t_i) = \int_{t_i}^{t_{i+1}} \phi(t_{i+1}, t)\,dv(t)$$

11

Sampling 2.

$$y(t_{i+1}) = y(t_i) + \int_{t_i}^{t_{i+1}} C x(t)\,dt + \int_{t_i}^{t_{i+1}} de(s)$$

$$= y(t_i) + \int_{t_i}^{t_{i+1}} C\phi(s,t_i)\,\underline{x(t_i)}\,ds$$

$$+ \int_{t_i}^{t_{i+1}} C\left[\int_{t_i}^{s} \phi(s,t)\,dv(t)\right] ds + \int_{t_i}^{t_{i+1}} de(s)$$

$$= y(t_i) + \Theta(t_{i+1}, t_i)\,x(t_i) + \tilde{e}(t_i)$$

$$\tilde{e}(t_i) = \int_{t_i}^{t_{i+1}} \Theta(t_{i+1}, t_i)\,dv(t) + e(t_{i+1}) - e(t_i)$$

Vilka egenskaper har

$\tilde{v}(t_i)$ och $\tilde{e}(t_i)$???

$$\int_{t_i}^{t_{i+1}} C \int_{t_i}^{s} \phi(s,t)\,dv(t)\,ds = \int_{t_i}^{t_{i+1}} \left(\int_{t}^{t_{i+1}} C\phi(s,t)\,ds \right) dv(t)$$

$$= \int_{t_i}^{t_{i+1}} \Theta(t_{i+1}, t)\,dv(t)$$

12

Sampling 3

$$E\,\tilde{v}(t_i) = E\,\tilde{e}(t_i) = 0$$

$$E\,\tilde{v}\tilde{v}^T = \int \phi R_1 \phi^T\,dt$$

$$E\,\tilde{e}\tilde{e}^T = \int \Theta R_1 \Theta^T dt + \int_{t_i}^{t_{i+1}} R_2\,dt$$

$$E\,\tilde{v}\tilde{e}^T = \int \phi R_1 \Theta^T\,dt$$

OBS oftas $\neq 0$

Gäller för tidsberoende

A, C, R_1 och R_2