Jitterbug – A Matlab Toolbox for Stochastic Control Performance Analysis

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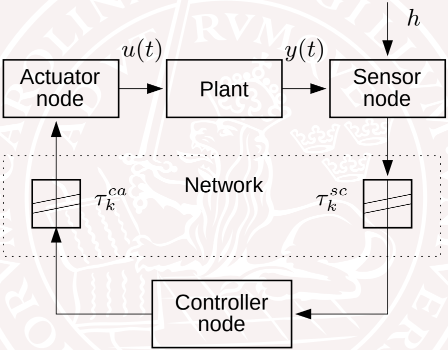
Motivation 1: Networked Control

Johan Nilsson: *Real-Time Control Systems with Delays*, PhD thesis, 1998 (cited 1450 times)

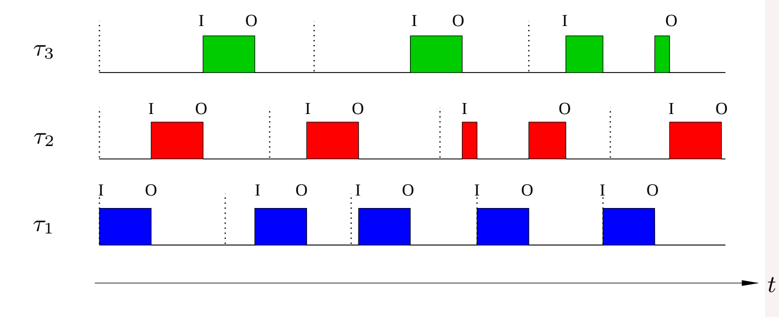


How is performance affected by time-varying delays τ_k^{sc} and τ_k^{ca}?

How to design an optimal controller?

Motivation 2: Real-Time Scheduling

Example: Three control tasks executing on a shared computing platform under EDF scheduling



How is performance affected by input and output jitter?

How to design an optimal controller?

Analysis Using Jitterbug

Inputs:

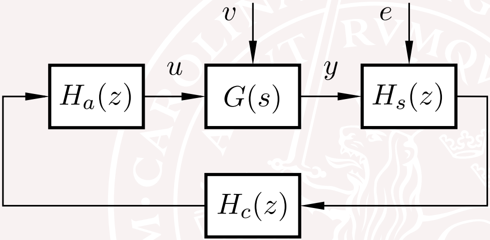
- Signal model
  - Continuous-time and discrete-time linear systems driven by white noise
  - Quadratic cost functions
- Timing model
  - Determines when the discrete-time systems are updated
  - Basic period h
  - Random delays between timing nodes described by probability mass functions with resolution δ

Outputs:

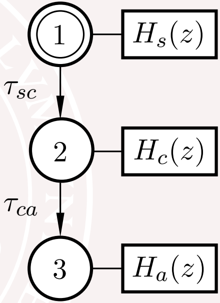
- Total cost (∞ if not mean-square stable)
- Spectral densities of all outputs

Example: Networked Control Loop

Signal model

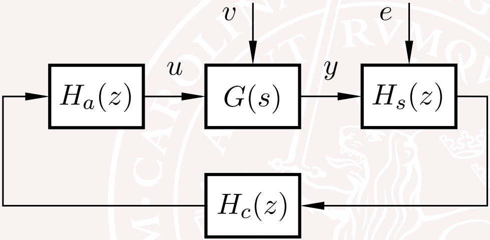


Periodic timing model

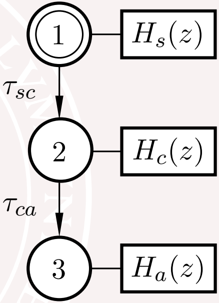


Example: Networked Control Loop

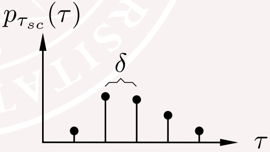
Signal model



Periodic timing model



Example of probability mass function:



Example of Analysis

Plant: $G(s) = \frac{1000}{s(s+1)}, \; R_{1c} = 1$

Sampler: $H_s(z) = 1, \qquad R_2 = 0.01$

Controller: $H_c(z) = -K\left(1 + \frac{T_d}{h}\frac{z-1}{z}\right)$

Actuator: $H_a(z) = 1$

Cost function: $J = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(y^2(t) + u^2(t)\right) dt$

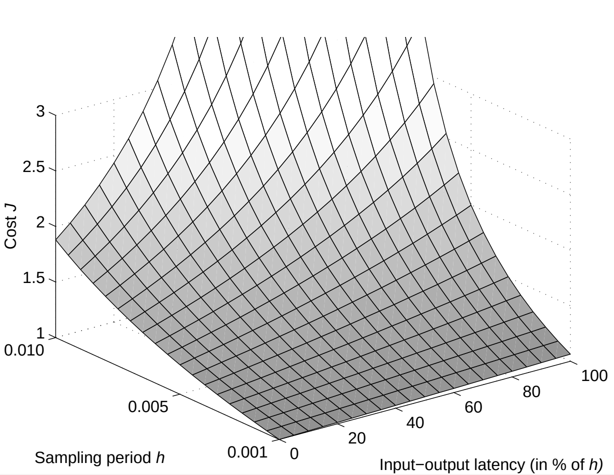
Delay distributions: $p_{\tau_{sc}}, p_{\tau_{ca}}$

Matlab Commands

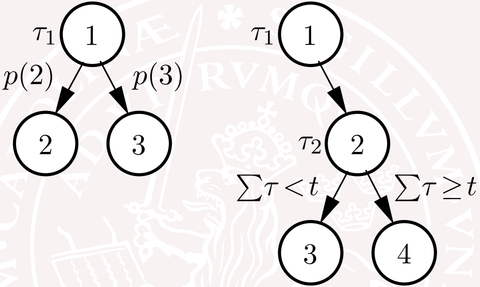
```
G = 1000/(s*(s+1));
Qc = diag([1 1]);
Hs = 1;
Hc = -K*(1+Td/h*(z-1)/z);
Ha = 1;
Ptausc = [ ... ];
Ptauca = [ ... ];
N = initjitterbug(delta,h);
N = addtimingnode(N,1,Ptausc,2);
N = addtimingnode(N,2,Ptauca,3);
N = addtimingnode(N,3);
N = addcontsys(N,1,G,4,Qc,R1c);
N = adddiscsys(N,2,Hs,1,1,[],R2);
N = adddiscsys(N,3,Hc,2,2);
N = adddiscsys(N,4,Ha,3,3);
N = calcdynamics(N);
J = calccost(N)
```

Example of Results

Vary sampling period h and total delay:



More Complicated Models



- random choice of path
- choice of path depending on previous delay
- different update equations in different nodes
- different update equations depending on previous delay
- aperiodic systems

Internal Workings

1. Sample the continuous-time systems, the continuous-time noise, and the cost functions with the time-grain δ
2. Translate the timing model into a Markov chain
3. Formulate the closed-loop system as a **jump linear system**

$$x(k+1) = \Phi(m)x(k) + e(k), \quad \mathbf{E}\left\{e(k)e^T(k)\right\} = R(m)$$

   where $\Phi(m)$ and $R(m)$ depend on the Markov state m
4. Compute the stationary covariance $P = \mathbf{E}\left\{xx^T\right\}$ from

$$P(k+1) = \mathbf{E}\left\{\Phi(m)P(k)\Phi(m)^T + R(m)\right\}$$

   - Aperiodic systems: iterate until convergence
   - Periodic systems: solve system of n^2 linear equations

Control Design Using Jitterbug

Sampled-data LQG controller for continuous-time plant

- ZOH or impulse hold sampling
- Kalman filter with or without direct term
- Constant-delay design
- Variable-delay design $\Rightarrow$ stochastic Riccati equation

Example: Intersample Variance

[See *Computer-Controlled Systems*, p. 493–495]

Consider ZOH control of

$$dx = u\,dt + dv$$

where $v(t)$ is a Wiener process with incremental covariance dt.

Assume sampled measurements

$$y(t_k) = x(t_k) + \varepsilon(t_k), \quad \mathbf{E}\,\varepsilon^2 = 1$$

Example: Intersample Variance

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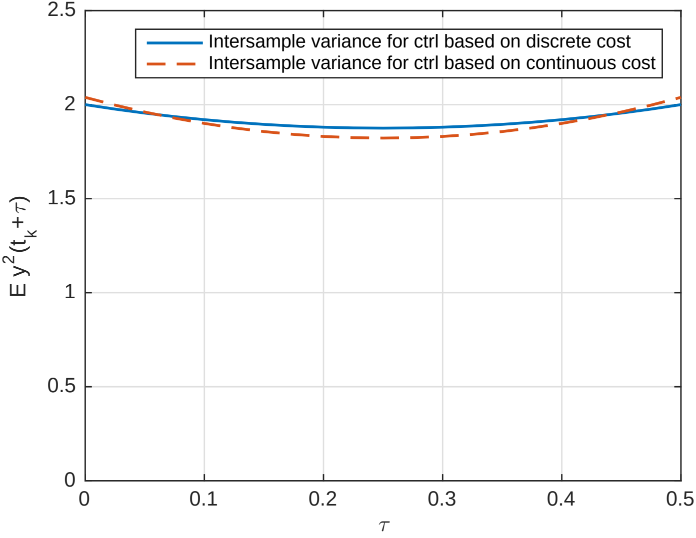
Compare:

- Controller minimizing $J = \mathbf{E}\,y^2(t_k)$
- Controller minimizing $J = \mathbf{E}\int_0^h (y^2(t_k+\tau))d\tau$

Example: Intersample Variance



Example: Delay Compensation

Plant: $P(s) = \dfrac{1}{s^2 - 1},\ R_{1c} = 1$

Sampler: $h = 0.5,\qquad R_2 = 0.01$

Cost function: $J = \mathbf{E}\big(y^2(t) + 0.01u^2(t)\big)\,dt$

Input-output delay: $\tau \in U(0, \tau_{\max}),\ 0 \le \tau_{\max} \le h$

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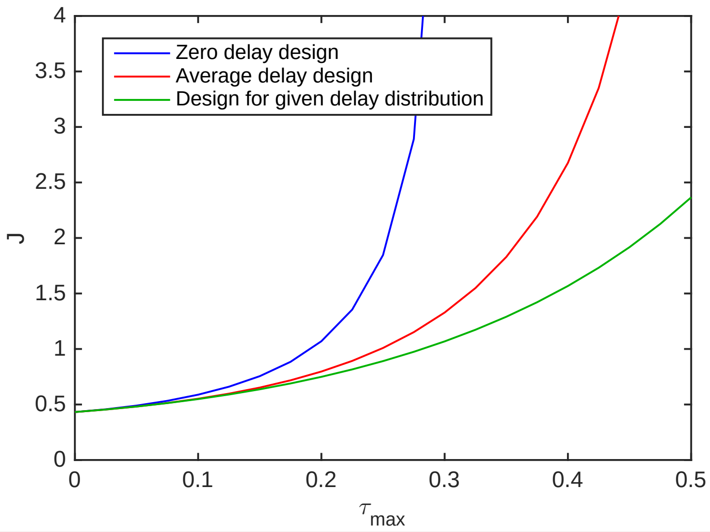
Compare:

- Controller designed assuming zero delay
- Controller designed for average delay
- Controller designed for the given delay distribution

Example: Delay Compensation



Conclusions

- Jitterbug can evaluate quadratic performance criteria for linear stochastic systems under quite general timing models
  - Input-output delay
  - Sampling and output jitter
  - Vacant sampling, lost controls
  - Gain scheduling based on actual delay
  - . . .
- Convenient commands for delay-aware LQG design
- Limitations:
  - Average-case performance analysis (no worst-case guarantees)
  - Delays assumed independent from period to period

References

Y. Ji, H. J. Chizeck, X. Feng & K. A. Loparo (1991): “Stability and control of discrete-time jump linear systems.” *Control-Theory and Advanced Applications*, 7:2, pp. 447–270.

J. Nilsson (1998): *Real-Time Control Systems with Delays*. PhD Thesis TFRT-1049, Department of Automatic Control, Lund Institute of Technology.

B. Lincoln & A. Cervin (2002): “Jitterbug: A Tool for Analysis of Real-Time Control Performance”. In *Proc. IEEE Conference on Decision and Control*.

Download:

`http://www.control.lth.se/research/tools/jitterbug/`