Exercise 7

1. Problem 16.5 in the course book

2. Problems 16.8 and 16.9 in the course book. For each of these problems plot on the same figure: 1) Bode diagram of WP; 2) Bode diagram of the open loop $PK_\infty W$.

   (The notation b_{opt} in the book is parallel to the notation α_{max} in the lecture slides)

3. Consider a simplified model of a satellite with two highly flexible solar arrays (Salehi, 10th IFAC Symposium, 1985)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & -2\zeta\omega \end{bmatrix} x + \begin{bmatrix} 0 \\ 1.732 \cdot 10^{-5} \\ 0 \\ 3.786 \cdot 10^{-4} \end{bmatrix} u + \begin{bmatrix} 0 \\ 1.732 \cdot 10^{-5} \\ 0 \\ 3.786 \cdot 10^{-4} \end{bmatrix} v$$

   where $\omega = 1.539$ rad/sec is the frequency of the flexure mode and $\zeta = 0.003$ is the flexural damping ratio. Here u is the control torque (Nm), v is a constant disturbance torque (Nm) and y is the roll angle measurement (rad).

   Required performance specification: y must stay within $0.04° \approx 0.0007$ rad pointing accuracy due to 0.3 Nm step torque disturbances.

   - Using [Zhou,16.12] prove the following bound

$$\left|(I - PK)^{-1}P\right| \leq \frac{\gamma}{|W|},$$

     where $W = W_1W_2$ is a single shaping function (for SISO plants).

   - Show that the performance specification is met for the constant shape function $W = 2000$.

   - Find the controller by H^∞ loop shaping procedure. Draw the Bode plots of the nominal P, shaped P_s and achieved open loop. Compare.

   - Due to the steady-state error ($\approx 0.018°$) introduce another shaping function

$$W = \frac{10000(s + 0.4)}{s}.$$

     Show that the performance specification is met and the full steady-state disturbance rejection is guaranteed.

   - Repeat the third item for the new W.

4. Problem 16.3 in the course book. (This is an optional problem for those who are interested in detailed understanding of the derivations in Chapter 16.)