Exercise 5

1. † Prove the third row in the table on page 19 of Lecture 5. (Note that the formulae in this table correspond to stabilization with *negative* feedback.)

2. Prove the last row in the table on page 19 of Lecture 5.

3. Consider a plant with additive uncertainty $P_\Delta = P_0 + W\Delta$ for $\Delta \in \gamma \cdot \mathcal{B}RH_\infty$, where γ is the radius of the admissible uncertainty,

$$P_0 = \frac{1}{s-1} \quad \text{and} \quad W = \frac{s+0.01}{s+1}.$$

   Find a stabilizing controller that maximizes the uncertainty radius γ.

4. † Problem 8.1 in the course book.

5. Consider the family of plants $P(s,h) = \frac{1}{s}e^{-hs}$ with $h \in [0,1]$. In this problem we are going to bound $P(s,h)$ using the additive uncertainty model $P_\Delta = P_0 + W\Delta$ for $\Delta \in \mathcal{B}RH_\infty$.

   - Choose $P_0(s) = \frac{1}{s}$ as a nominal plant. Find an explicit expression for

   $$R(w) = \max_{h\in[0,1]} (|P(iw,h) - P_0(iw)|)$$

     and calculate $\lim_{w\to 0} R(w)$.
   - Draw schematic representation of the family of plants $P(s,h)$ on the Nyquist diagram.
   - Construct a first-order uncertainty weight W.
   - Propose a way for constructing a more tight additive uncertainty model. (Do not calculate anything, just give an idea.)

6. Problem 8.12 in the course book.