Exercise 4

1. † Consider the generalized plant

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{s-\alpha+1}{s-\alpha} \\ \frac{s+2}{s+1} & -\frac{1}{s-\alpha} \end{bmatrix} = \left[\begin{array}{cc|cc} \alpha & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{array}\right]$$

for $\alpha \in (0, 1)$.

- Verify that $X = 0$ and

$$Y = \frac{2\alpha}{(1+\alpha)^2} \begin{bmatrix} (\alpha+2)^2 & \alpha+2 \\ \alpha+2 & 1 \end{bmatrix}$$

are the stabilizing solutions of the $\mathcal{H}^2$ Riccati equations.
- Calculate the optimal solution of the $\mathcal{H}^2$ problem.
- How does the achievable $\mathcal{H}^2$ performance depend on α?
- Consider the problem and the solution for $\alpha \to 1$.
Is the problem well posed? Is the solution stabilizing?

(It is convenient to use Matlab symbolic toolbox to solve this problem.)

2. Show that

$$\left\| \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \right\|_\infty < 1 \quad \Rightarrow \quad ||G_1||_\infty < 1, \quad ||G_2||_\infty < 1,$$

but not the other way around.
(This fact produces conservatism in the formulation of the mixed sensitivity problem, see page 22 in Lecture 4.)

3. † Consider a plant $P(s) = \frac{1}{s^2+0.1s+1}$

- Formulate weighted sensitivity problem for $\epsilon_\sigma = 0.01$, $\omega_0 = 1$ and $\delta_\sigma = 0.01$. Choose the second order approximation of the weighting function.
- Solve the weighted sensitivity problem. Plot bode magnitude diagrams for $1/W_\sigma$, S_o and PS_o. (Place the first two on the same plot.) Plot the step response of the closed loop output and control signal.
- Complement the weighted sensitivity formulation from the previous questions to the mixed sensitivity problem. Use the weight $W_\chi = 0.25\frac{0.1s+1}{10^{-4}s+1}$.
- Solve the mixed sensitivity problem. Plot bode magnitude diagrams for $1/W_\sigma$, S_o and $1/W_\chi$, PS_o. Plot the step response of the closed loop output and control signal.

4. Problem 13.5 from the course book.

5. Problem 14.5 from the course book. For both 14.5 and 13.5:

   - Plot maximal singular value of $T_{\text{cl}}(jw)$ as a function of w, where $T_{\text{cl}}(s)$ is the optimal closed loop transfer matrix.
   - Plot impulse response of the optimal closed loop, smoothed by $\frac{1}{0.01s+1}$. Find the energy of this response.
   - Plot response of the optimal closed loop for $r(t) = \sin(10t)e^{-0.1t}$.

6. Problem 14.6 from the course book.

7. Problem 14.7 from the course book.