Exercise 3

1. Consider the definitions of upper and lower LFTs $\mathcal{F}_l(\cdot)$ and $\mathcal{F}_u(\cdot)$ in the beginning of Lecture 3. Prove that

   - $\mathcal{F}_u(\Phi, \Omega) = \mathcal{F}_l(\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right] \Phi \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right], \Omega)$
   - $\Theta = \mathcal{F}_l(\Phi, \Omega) \quad \Leftrightarrow \quad \Omega = \mathcal{F}_u(\Phi^{-1}, \Theta)$ (if Φ is invertible)

2. † Consider a measured disturbance attenuation problem depicted on Fig. 1, where P is the plant, K is a feedback controller and W_* are the weights for the external signals. The aim is to lessen the influence of the disturbance d on the output y, while keeping the control effort u not too large. The measurement of d corrupted with the noise n is available to the controller.

   - Construct a generalized plant for the problem.
   - Construct a generalized plant for the problem with an additional constraint on the feedback part of the controller to contain integrator.

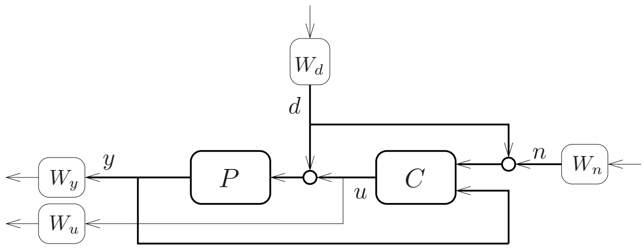

Figure 1: Measured disturbance rejection

3. Prove that if the state-space conditions for stabilizability (page 16 in Lecture 3) hold, then the problem is indeed stabilizable. (Hint: show that lcf of a required form can be constructed for the generalized plant.)

4. † Consider the following three generalized plants:

$$G_i = \left[\begin{array}{c:c} \frac{1}{s+5} & 3 \\ 0 & 1 \\ 1 & \frac{1}{s+5} \\ \hdashline 2 & \frac{1}{s+5} \end{array}\right], \quad G_{ii} = \left[\begin{array}{c:c} \frac{1}{s+5} & 3 \\ 0 & 1 \\ 1 & \frac{1}{s-5} \\ \hdashline 2 & \frac{1}{s-5} \end{array}\right], \quad G_{iii} = \left[\begin{array}{c:c} \frac{1}{s-5} & 3 \\ 1 & \frac{1}{s-5} \\ 5 & 0 \\ \hdashline 5 & \frac{1}{s-5} \end{array}\right].$$

   Are they internally stabilizable? If yes, parametrize all stabilizing controllers and find, if exists, one Q for which the controller is static.

5. Consider the stabilization problem for the plant $P(s) = \frac{1}{s-1}$ using positive feedback.

- Construct a doubly coprime factorization of $P(s)$ for which all eigenvalues of $A+BF$ and $A+LC$ are at -1. Using this factorization find the parametrization of all stabilizing controllers for $P(s)$.
- The same as in the previous item but with eigenvalues at -2.
- Obviously, the static controller $K(s) = -k$ stabilizes $P(s)$ for all $k > 1$. For each of the parametrizations, find the parameters $Q(s)$ producing this static controller.

6. Consider a 2DOF control problem depicted in Fig. 2 (left). Two groups of engineers decided to work on this problem independently. The first group will use classical methods to design feedback controller K for the setting on Fig. 2 (right). Their aim is to guarantee internal stability and disturbance rejection. The second group will address tracking behavior via minimization

$$\min \left\| \begin{bmatrix} I \\ 0 \end{bmatrix} + \begin{bmatrix} -N \\ M \end{bmatrix} Q_1 \right\|_2 ,$$

where $P = NM^{-1}$ is the rcf of the plant and Q_1 is the first part of the Youla parameter, see page 23 in Lecture 3.

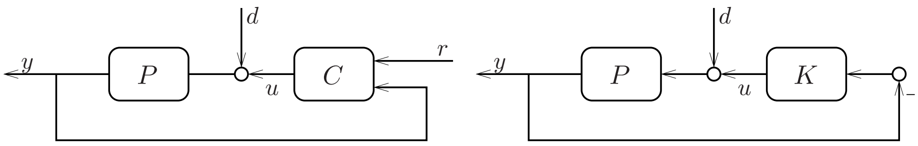

Figure 2: 2DOF tracking

- Express the resulting controller C in terms of K and Q_1
- They decided to implement the controller as shown on Fig 3. What is the natural choice for X_1, X_2 and X_3?
(Express in terms of K and Q_1.)

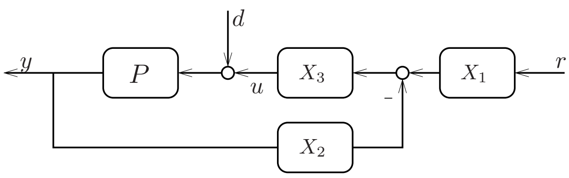

Figure 3: 2DOF controller implementation