Optimization with CasADi, December 2011

Hand-in exercises

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Problem 1

Consider a nonlinear pendulum given by the dynamics:

$$\dot{p} = v \tag{1}$$

$$\dot{v} = u - C\,\sin(\frac{p}{C}) \tag{2}$$

with $C = 18/\pi$. Define the state vector $x = [p, v]^{\mathrm{T}}$.

Use the RK4 integrator from exercise 2,

$$k_1 = f(x_k, u_k) \tag{3}$$

$$k_2 = f(x_k + \frac{1}{2}\,\Delta t\, k_1, u_k) \tag{4}$$

$$k_3 = f(x_k + \frac{1}{2}\,\Delta t\, k_2, u_k) \tag{5}$$

$$k_4 = f(x_k + \Delta t\, k_3, u_k) \tag{6}$$

$$x_{k+1} = x_k + \frac{1}{6}\,\Delta t\,(k_1 + 2\,k_2 + 2\,k_3 + k_4) \tag{7}$$

to define a discrete time system of the form $x_{k+1} = \Phi(x_k, u_k)$ (only one RK4 step per control interval).

Choose the initial values $p_0 = 10$, $v_0 = 0$ and take $N = 50$ time steps, each of size $\Delta t = 0.2$. Constrain p and v to the interval $[-10, 10]$ and u to the interval $[-3, 3]$. This allows us to formulate the following discrete time optimal control problem:

$$\begin{array}{lrl} \underset{x_0, u_0, x_1, \ldots, u_{N-1}, x_N}{\text{minimize:}} & \displaystyle\sum_{k=0}^{N-1} \|u_k\|_2^2 & \\ & [p_0, v_0]^{\mathrm{T}} - x_0 = 0 & \\ & \Phi(x_k, u_k) - x_{k+1} = 0 & \text{for } k = 0, \ldots, N-1 \\ \text{subject to:} & x_N = 0 & \\ & -10 \le x_k \le 10 & \text{for } k = 0, \ldots, N-1 \\ & -10 \le u_k \le 10 & \text{for } k = 0, \ldots, N-1 \end{array} \tag{8}$$

Solve the problem using CasADi and IPOPT and plot the optimal solution.

Problem 2

Formulate the problem in Modelica and Optimica and solve the problem with JModelica.org. Use the Radau discretization method that utilizes CasADi as demonstrated in the JModelica.org exercise session.

Compare the results and discuss:

- Are the solutions the same if 50 finite elements are used in both algorithms? If not, why do they differ?
- Modify the optimal control problem by changing the initial conditions of the pendulum angle to the downward position. Can you solve the problem?
- Vary the number of finite elements. What happens if you use too few elements? Hint: use the optimal control profiles to drive a simulation of the system as demonstrated during the JModelica.org exercise session.
- What happens if you increase the number of elements by one or two magnitudes? Can you still solve the problem? Discuss the trade-off between accuracy and number of elements.