Network Dynamics: an Overview

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Network Dynamics, Ph.D. course, 9 ECTS

- 18 two-hours meetings (including today)
- 1 lecture + 1 exercise session per week
- (bi-)weekly homeworks + small final presentation
- reference books:

0 notes and specific research papers

1 Easley & Kleinberg, 'Networks, crowds, and markets', 2010

2 Newman, 'Networks', Oxford U.P., 2010

3 Levine, Peres & Wilmer, 'Markov chains and mixing', 2008

4 Draief & Massoulie, 'Epidemics and rumors in complex networks'

5 Aldous& Fill, 'Reversible Markov chains and random walks'

- today: broad intro, course overview

Similar courses around the world

- MIT grad: http://stellar.mit.edu/S/course/6/sp11/6.986/
- MIT undergrad: http://stellar.mit.edu/S/course/6/fa09/6.207J/
- Cornell undergrad (by D.Easley, J.Kleinberg, & E.Tardos):: http://www.infosci.cornell.edu/courses/info2040/2011fa/
- Berkeley grad 1 (by D.Aldous): http://www.stat.berkeley.edu/ aldous/260-FMIE/index.html
- Berkeley grad 2 (by E.Mossel): http://www.stat.berkeley.edu/ mossel/teach/SocialChoiceNetworks10/index.html
- North-Easter U. (by A.Barabasi): http://barabasilab.neu.edu/courses/phys5116/
- Michigan U. (by L.Adamic): http://open.umich.edu/education/si/si508/fall2008/materials

(Complex) networks

(Large-scale) systems of (simple) interacting units

- infrastructure networks: transportation, power, gas, and water distribution, sewer, Internet
- informational networks: WWW, citation networks
- social networks: friendships, family ties, Facebook etc.
- economic networks: supply chains
- financial networks: borrowing-lending nets
- biological networks: neural networks, gene/protein interactions
- ecological networks: food webs, flocks, ...

Studying (complex) networks

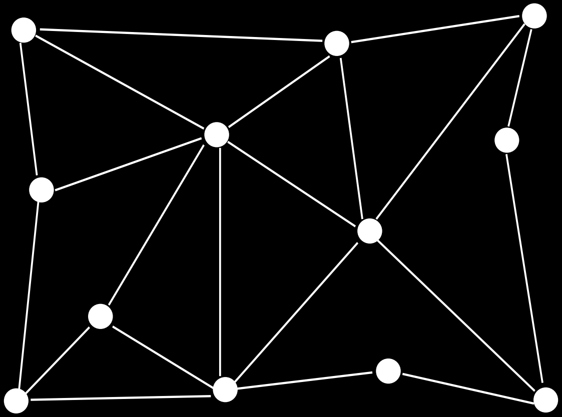
network structure + interaction mechanism

$\Downarrow$

emerging behavior

- spread of epidemics and information
- design of distributed algorithms
- opinion formation, social influence, and learning
- network robustness,
- cascaded failures, systemic risk
- ...

Mathematical representation of network structure

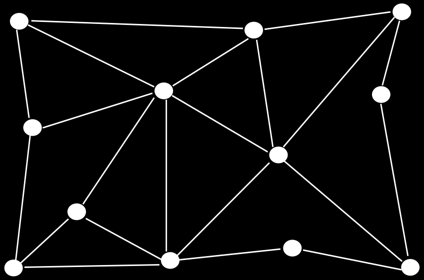


(un)directed (weighted) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

$\mathcal{V} =$ set of vertices (or nodes) $\qquad n = |\mathcal{V}| < +\infty$

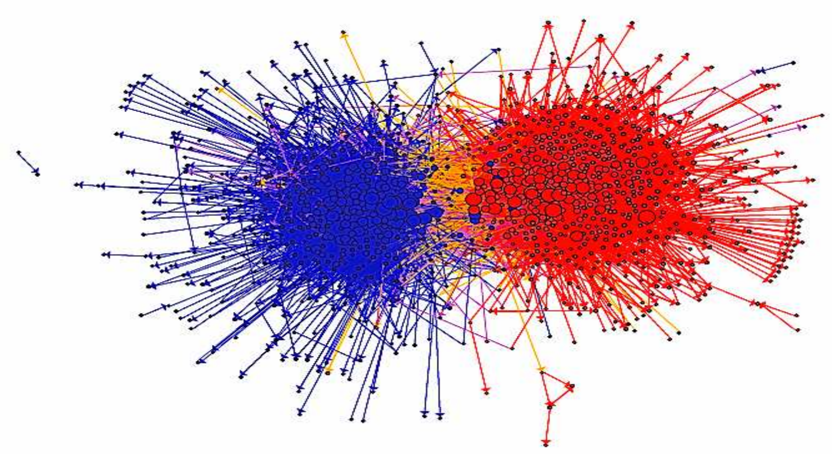
$\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} =$ set of edges (or links)

Examples 1



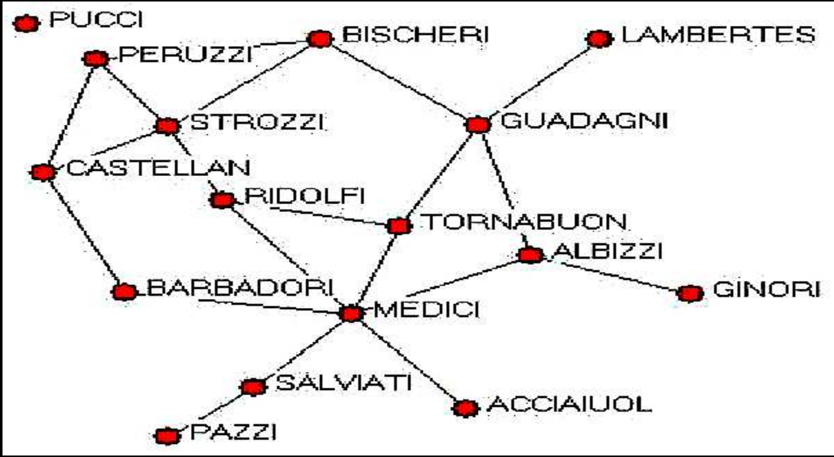
- Internet: nodes=routers, edges=direct physical links (und.)
- traffic networks: nodes=junctions, links=roads (directed)
- actors collaboration: nodes=actors, link⇔ same movie (und.)
- scientific collab.: nodes=researchers, link=coauthors (und.)

Example 2: political blogs before 2004 US elections



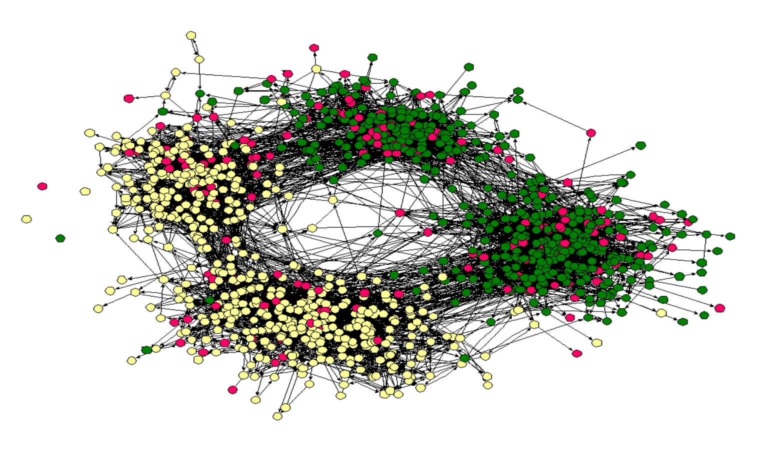
from Adamic and Glance, 'The Political Blogosphere and the 2004 U.S. Election: Divided They Blog', 2005

Example 3: Family ties in 15th century Florence



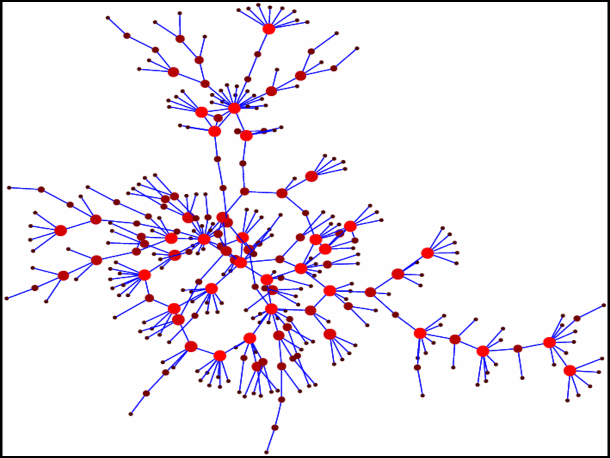
from Padgett and Ansell, ‘Robust action and the rise of the Medici, 1400-1434’, 1993

Example 4: High school friendships



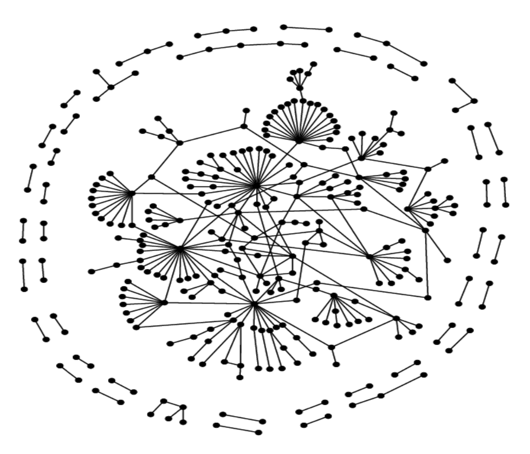
from Moody, ‘Race, school integration, and friendship segregation in America’, 2002

Example 5: Sexual contacts



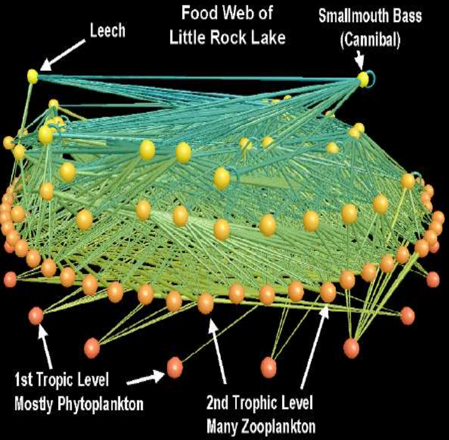
from Newman, 'The structure and function of complex systems', 2003

Example 6: protein network in yeast nucleus



from Maslov and Sneppen ‘Specicity and stability in topology of protein networks’, 2002

Example 7: Freshwater food web



from Martinez, ‘Artifacts or attributes? Effects of resolution on the Little Rock Lake food web’, 1991

Random networks

network structure + interaction mechanism

$$\Downarrow$$

emerging behavior

structure of large-scale networks is difficult to describe exactly: huge or non directly accessible data

Random networks

network structure + interaction mechanism

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emerging behavior

structure of large-scale networks is difficult to describe exactly: huge or non directly accessible data

aggregate properties:

- connectivity, diameter / average distance
- frequency of subgraphs
- degree distribution

statistical approach:

- ensemble of graphs
- typical properties as $n = |\mathcal{V}| \to +\infty$

Complex networks

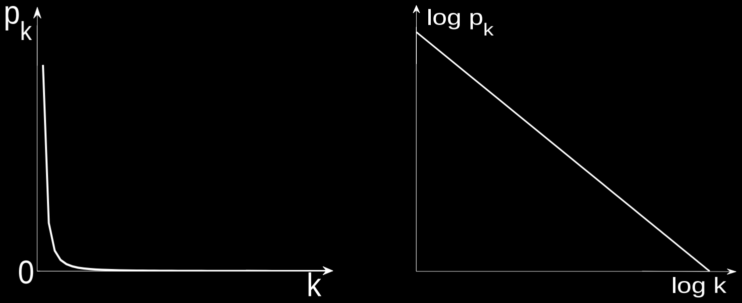
Properties widely observed in empirical studies

1. small world $\leftrightarrow$ diameter $\approx \log n$
2. high clustering $\leftrightarrow$ many triangles
3. scale free $\leftrightarrow$ power law degree distribution

Small world

- Milgram's experiment ('67): randomly selected group of few hundreds of people from Omaha (NE). A letter given to each of them to be delivered to a stock broker living in Boston (MA). Letter can only be handed to a person know directly. $\Longrightarrow$ 35% letters reach destination, median # of steps: 5.5 $\Longrightarrow$ "6 degrees of separation"
- Albert, Jeong, and Barabasi ('99): WWW network, $n \sim 800M$ average distance of webpages $\sim 0.35 + 2.06 \log n = 18.59$

Power laws



$$p_k \sim C k^{-\gamma}$$

show up in quite different contexts:

- percentage of words in a book
- percentage of cities of a given size
- percentage of people having a certain income

Power law $\Longrightarrow$ heavy tails: lots of large cities, lots of rich people

Typically explained by rich-gets-richer mechanisms

Power law networks

$$d_v := \#\{\text{neighbors of } v\} \qquad p_d := \frac{1}{n}\#\{v : d_v = d\}$$

$$p_d \sim Cd^{-\gamma}$$

Empirical studies:

- Barabasi and Albert ('99): WWW has $\gamma_{in} \sim 2.1$, $\gamma_{out} \sim 2.7$
- Faloutos ('99): Internet $\gamma \sim 2.16$
- actor collaborations: $\gamma \sim 2.3$
- Redner ('98): citation network: $\gamma_{in} \sim 2.6$, $p_d^{out} \sim C\exp(-Kd)$
- Liljeros ('01): # sexual partners per year (in Sweden) $\gamma_{male} \sim 3.3$, $\gamma_{female} \sim 3.5$

$$2 < \gamma \leq 3 \quad \Longrightarrow \quad \langle d \rangle < +\infty \quad \langle d^2 \rangle = +\infty$$

$$\gamma \geq 3 \quad \Longrightarrow \quad \langle d \rangle < +\infty \quad \langle d^2 \rangle < +\infty$$

power law $\leftrightarrow$ scale free

Random graphs 1: Erdös-Rényi

$$\mathcal{G}(n,p) = (\mathcal{V},\mathcal{E}) \qquad |\mathcal{V}| = n$$

$$\mathbb{P}\left(\{v,w\} \in \mathcal{E}\right) = p \qquad \text{mutually independent}$$

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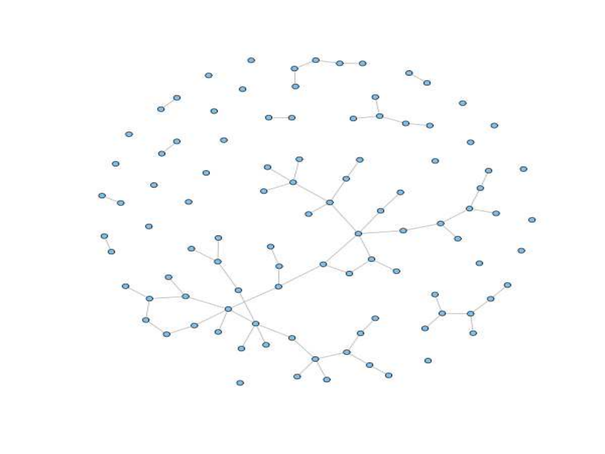

$n = 100$
$p = 0.15$

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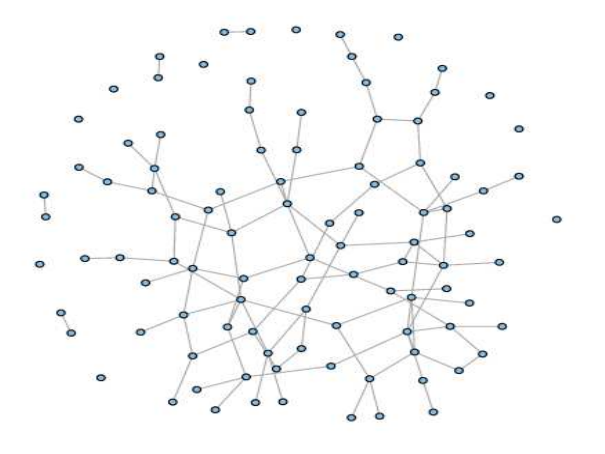

$n = 100$
$p = 0.2$

Random graphs 1: Erdös-Rényi

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If $p = \lambda/n$, with high probability as $n \to \infty$,

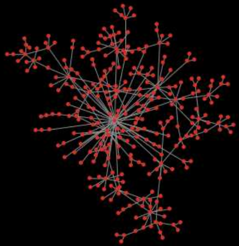
$$\text{phase transition} \quad \begin{cases} \lambda < 1 \Longrightarrow \text{size(largest component)} \asymp \log n \\ \lambda > 1 \Longrightarrow \left.\begin{array}{l} \text{size(largest component)} \asymp n \\ \text{diam(giant component)} \asymp \log n \end{array}\right\} \text{small world} \end{cases}$$

- Poisson degree distribution: $p_d \sim \dfrac{\lambda^d}{e^\lambda d!} \quad \Rightarrow \quad$ NO power law
- limited #(triangles) $\Rightarrow$ NO clustering

Random graphs 2: preferential attachment

Barabasi-Albert ('99)

1. start from a small given graph n_0
2. add a vertex and connect it with d older vertices randomly chosen with conditional probability $\propto$ their current degree
3. repeat step 2 $n - n_0$ times



http://ccl.northwestern.edu/netlogo/models/run.cgi?PreferentialAttachm

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$$\Longrightarrow \quad \begin{cases} p_d \sim Cd^{-3} \Rightarrow \text{power law} \\ diam \asymp \log n \Rightarrow \text{small world} \\ \text{(suitably modified)} \Rightarrow \text{high clustering} \end{cases}$$

Dynamics over networks

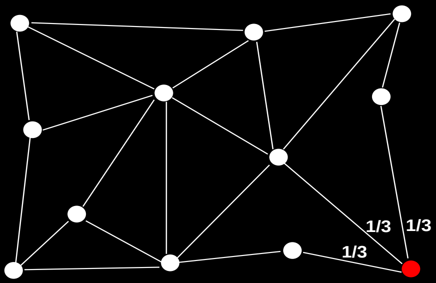
network structure + interaction mechanism

$\Downarrow$

emerging behavior

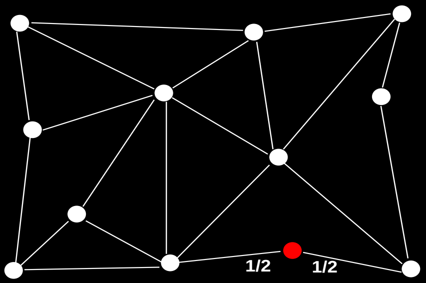
- random walks
- linear interactions: averaging / voter model
- epidemics: SI, SIR, SIS (contact model)
- other: majority model, evolutionary dynamics, games
- monotone dynamical systems
- ...

Random walk



- P stochastic matrix on $\mathcal{V}$, e.g., $P_{ij} = 1/d_i$
- $\mathbb{P}(V(t+1) = i | V(t) = i) = P_{ij}$
- network connected $\Rightarrow$ unique stationary distr. π $(\pi_v = \dfrac{d_v}{\sum_u d_u})$

Random walk



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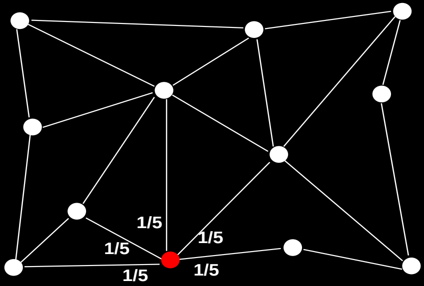

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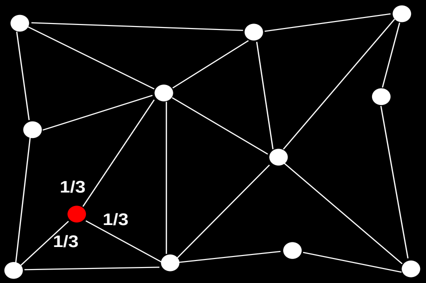

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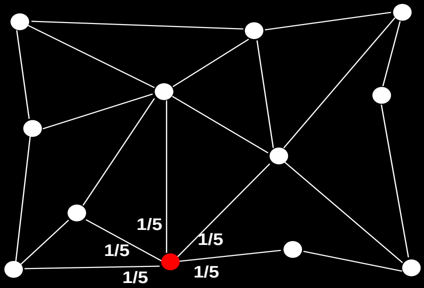

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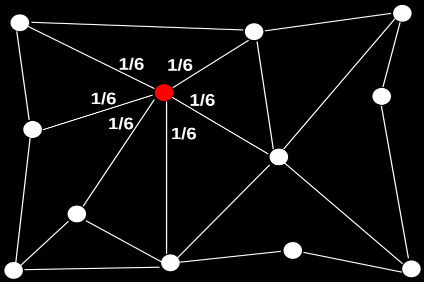

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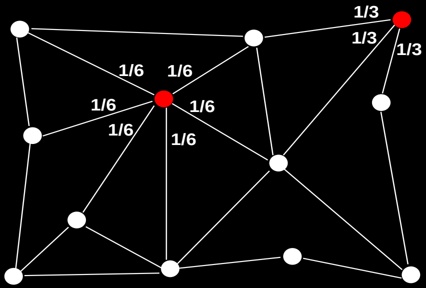
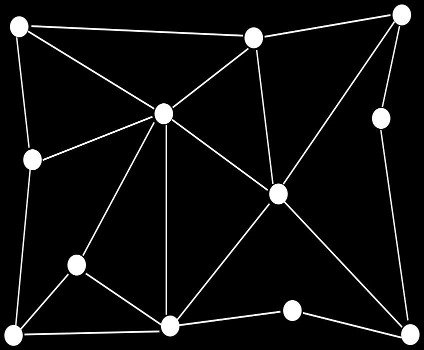
Random walk



- how fast does $||\mathbb{P}(V(t) = \cdot) - \pi||$ go to 0 ?
- when will $V(t)$ hit some other $w \in \mathcal{V}$?
- when will independent $V(t)$ and $\tilde{V}(t)$ meet for the first time?

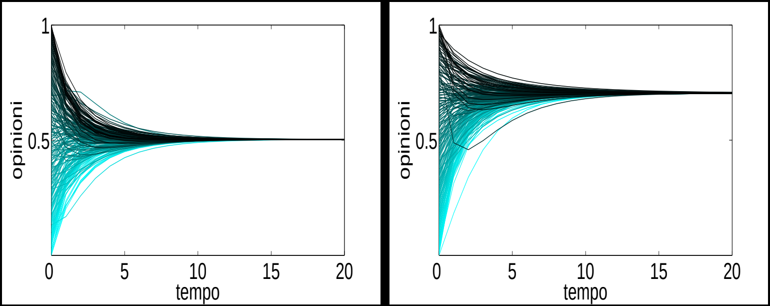
Distributed averaging



Gossip model:

- every node v has a state $x_v(t) \in \mathbb{R}$
- nodes get activated at independent Poisson times
- when a node v is activated, it choses a neighbor w at random and updates its value to $x_v(t) = (1-\omega)x_v(t^-) + \omega x_w(t^-)$
- network connected $\Longrightarrow$ convergence to consensus

Distributed averaging



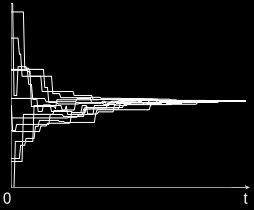
How is the limit consensus value related to the initial states of the nodes?
How does the network structure affect the speed of convergence?
What is the effect of heterogeneity of the agents behavior?

Consensus vs disagreement

most mathematical models:

connected network $\Longrightarrow$

(asymptotic) consensus



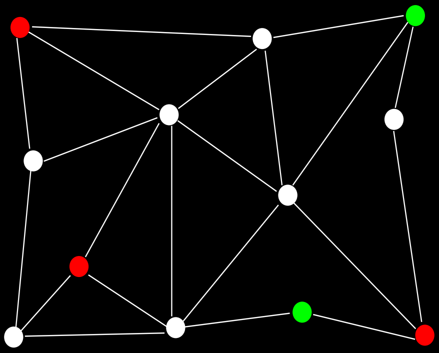
"Since universal ultimate agreement is an ubiquitous outcome of a very broad class of mathematical models, we are naturally led to inquire what on earth one must assume in order to generate ..." (Abelson '64)

"If people tend to become more alike in their opinions, attitudes, and behavior as they interact, why do not such differences eventually disappear?" (Axelrod '97)

Gossip model with stubborn agents

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ connected

opinions: $X_v(t) \in \mathbb{R}$



$\mathcal{V} = \mathcal{A} \cup \mathcal{S}$

$\mathcal{S} = \{$stubborn agents$\}$

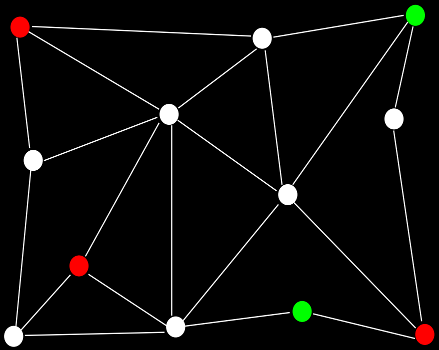
$X_s(t) = x_s \in [0, 1], \quad s \in \mathcal{S}$

model propaganda: political parties, media sources, advertising, ...

Gossip model with stubborn agents (cont'd)

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ connected

opinions: $X_v(t) \in \mathbb{R}$



$\mathcal{V} = \mathcal{A} \cup \mathcal{S}$

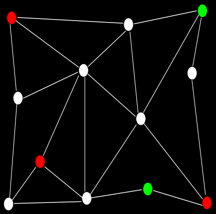
$\mathcal{A} = \{$regular individuals$\}$

$\omega \in]0,1]$ 'trust' $\qquad T^a :=$ random clock (rate-1 Poisson)

$$T^a_k = t \quad \Longrightarrow \quad a \text{ chooses } b \sim a \text{ at random}$$

$$X_a(t^-) = x\,,\ X_b(t^-) = y \quad \Longrightarrow \quad X_a(t) = x + \omega(y - x)$$

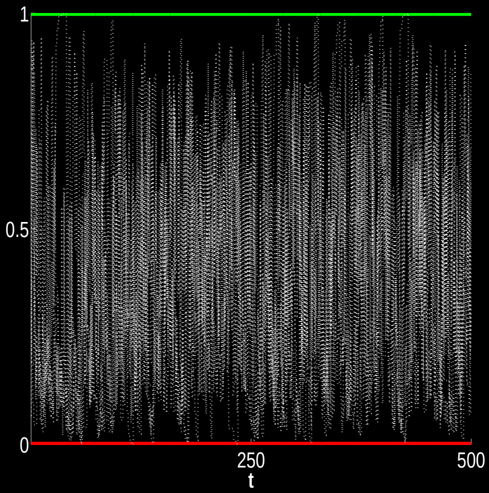
Typical sample-path behavior



$x_s = 1$

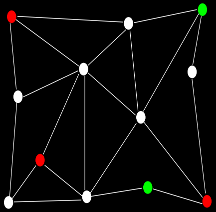
$x_s = 0$

$X_a(t)$



$$\exists s, s' \in \mathcal{S} : \ x_s \neq x_{s'} \quad \Longrightarrow \quad \mathbb{P}\,(\text{NO convergence, NO consensus}) = 1$$

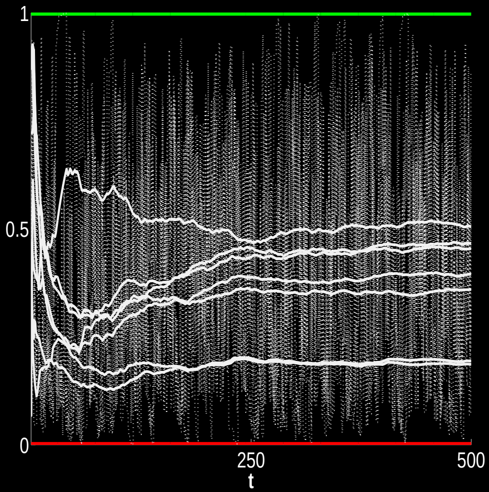
Typical sample-path behavior (cont'd)



$x_s = 1$

$x_s = 0$

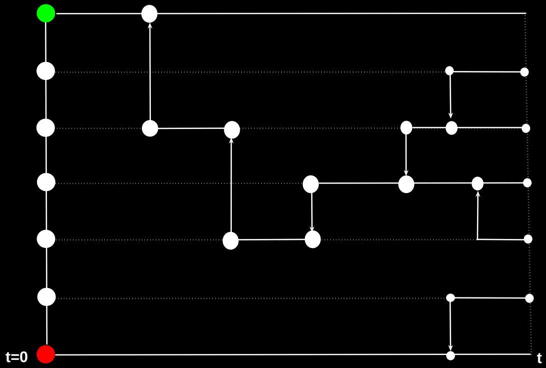
$Z_a(t) := \frac{1}{t}\int_0^t X_a(u)\mathrm{d}u$



$\mathbb{P}\left(Z_a(t) \text{ converges}\right) = 1$

Voter model

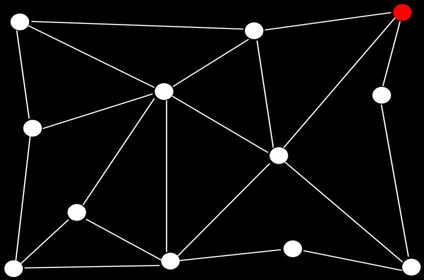
$\omega = 1 \Longrightarrow$ voter model: $\quad X_a(t) = X_b(t^-)$



Dual process: $(V_i(t))_{i\in\mathcal{V}}$ coalescing random walk, absorbing set $\mathcal{S}$

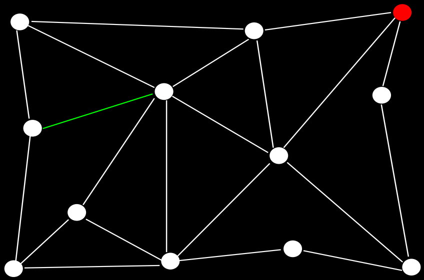
$$(X_i(t))_{i\in\mathcal{V}} \stackrel{d}{=} (X_{V_i(t)}(0))_{i\in\mathcal{V}}$$

Epidemics 1: SI



- S=susceptible I=infected
- $X_v(t) \in \{S, I\}$
- a random link gets activated at t (meeting)
- susceptible node becomes infected when meeting infected node

Epidemics 1: SI



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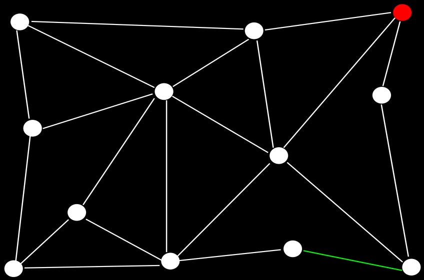

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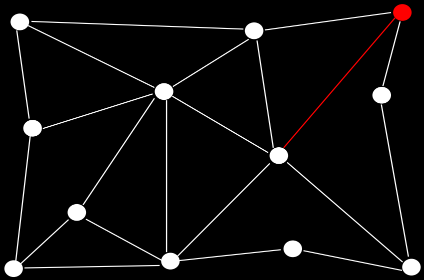

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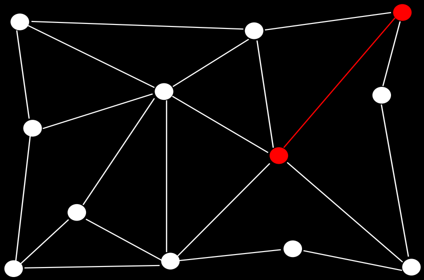

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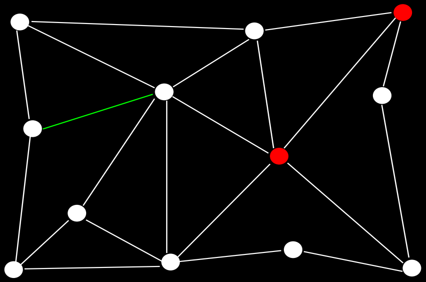

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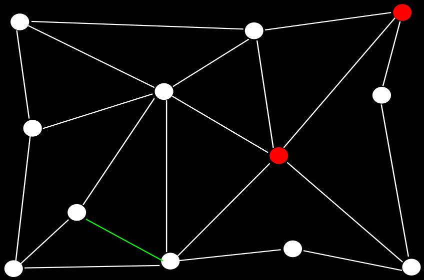

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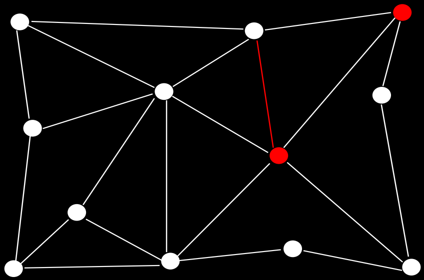

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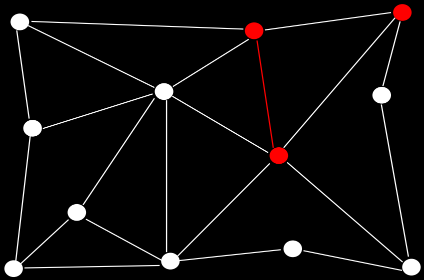

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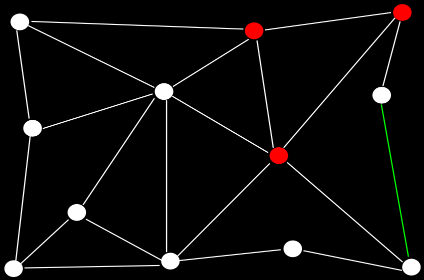

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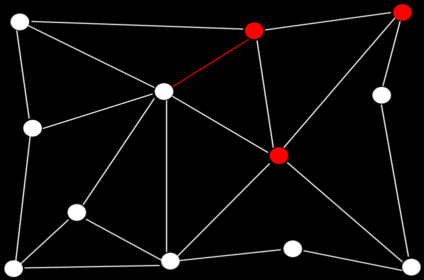

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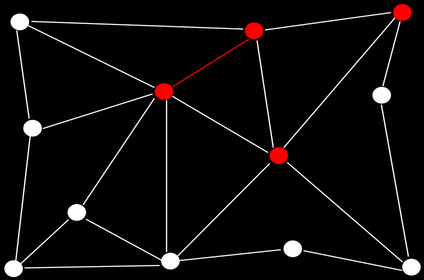

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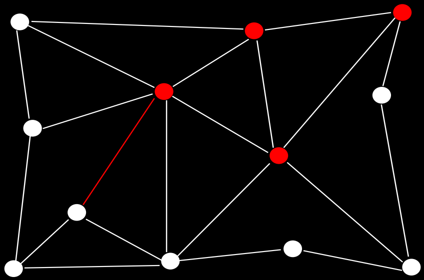

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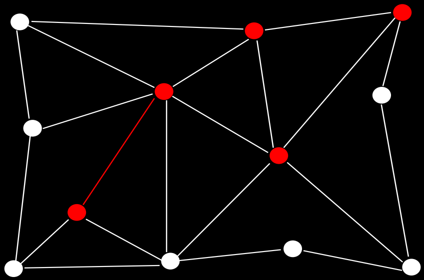

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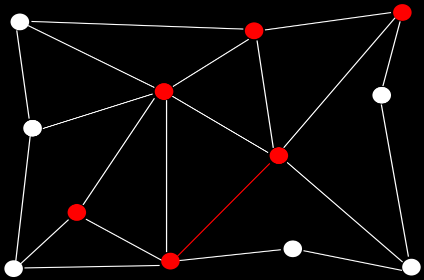

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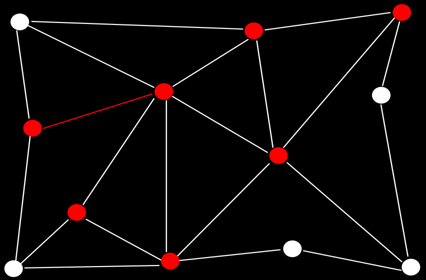

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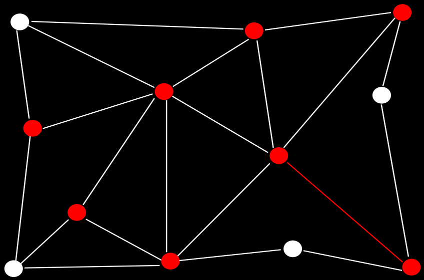

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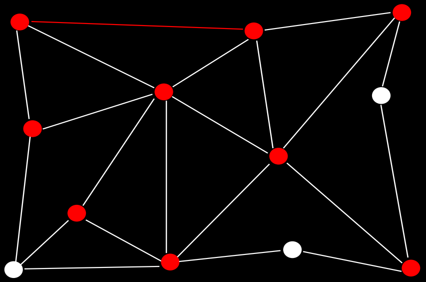

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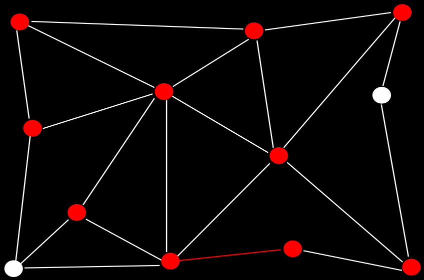

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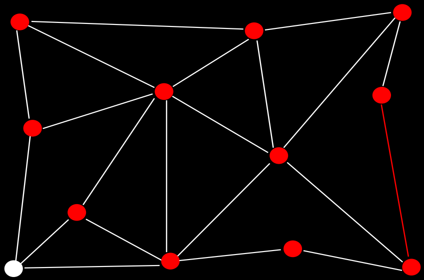

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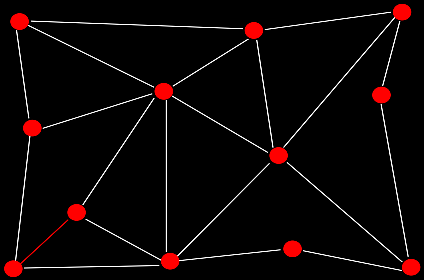
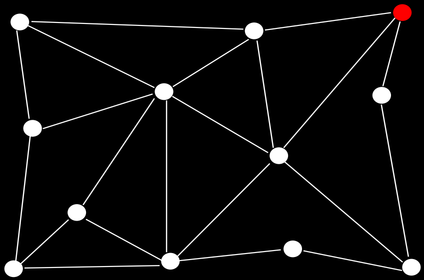


- $X_v(t) \in \{S, I\}$
- $x_v(t) \to I$ if and only if v is in the same connected component of some w with $x_w(0) = I$
- how fast will every node become infected?

Epidemics 2: SIR



- S=susceptible I=infected R=recovered
- $X_v(t) \in \{S, I, R\}$
- a random link gets activated at t (meeting)
- susceptible node becomes infected when meeting infected node
- random nodes get recovered (if infected)

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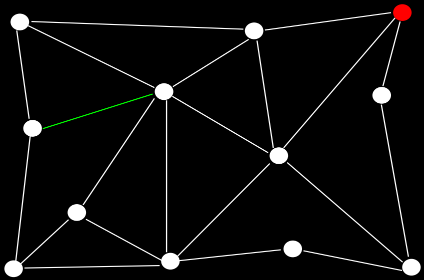

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- infected nodes recover (die) at random times

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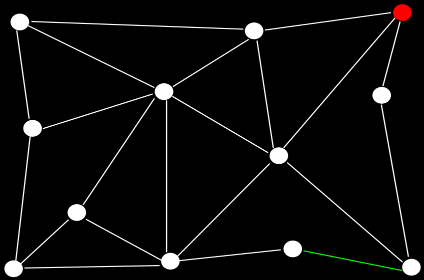

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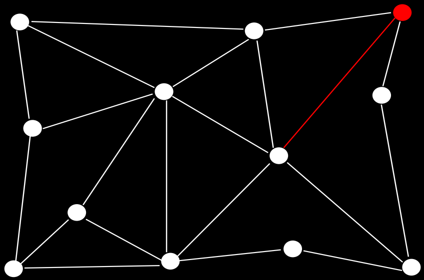

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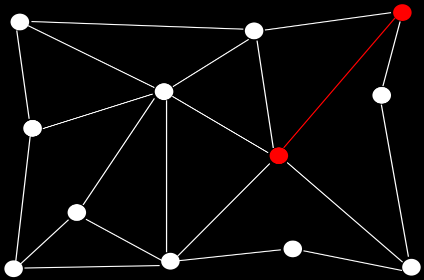

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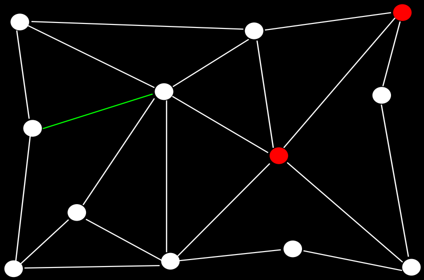

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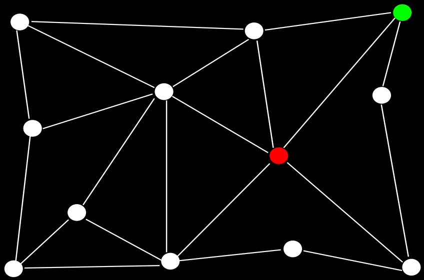

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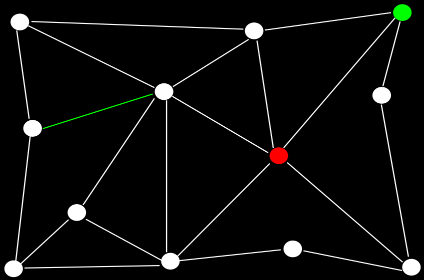

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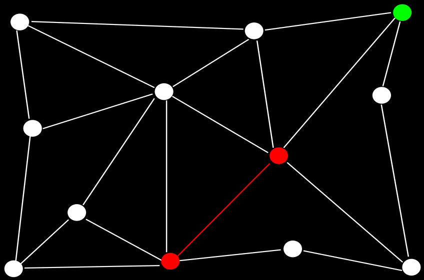

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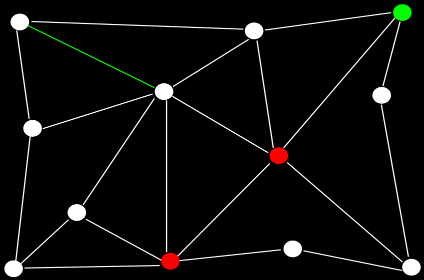

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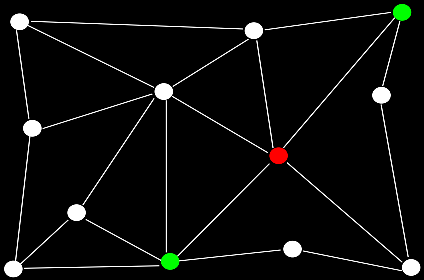

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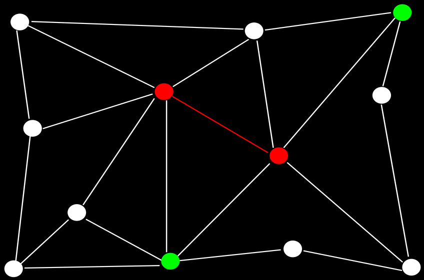

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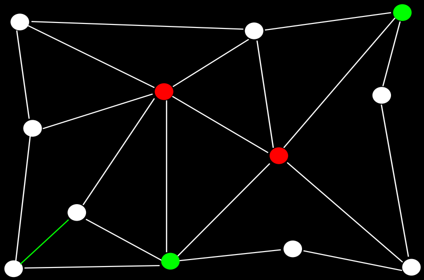

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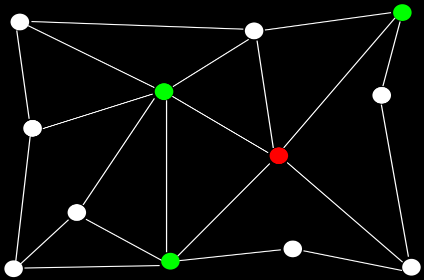

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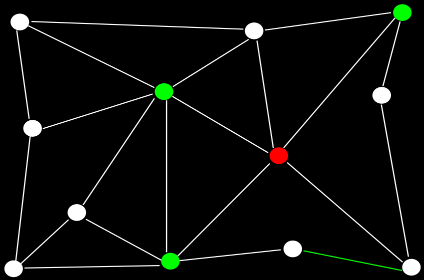

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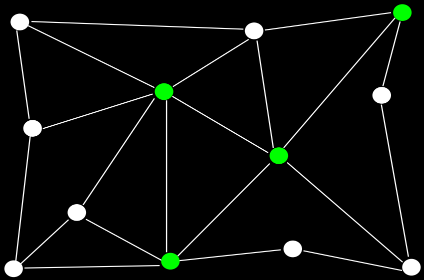
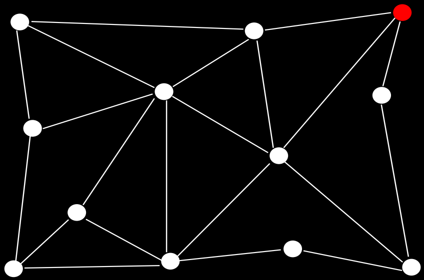


- S=susceptible I=infected R=recovered
- $X_v(t) \in \{S, I, R\}$
- the process stops in finite T with $x_v(T) \in \{I, R\}$
- how big is T?
- what is the fraction of $x_v(T) = R$?

Epidemics 3: SIS (a.k.a. contact model)



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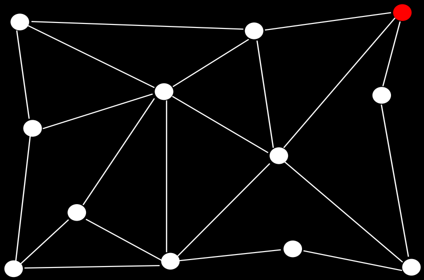

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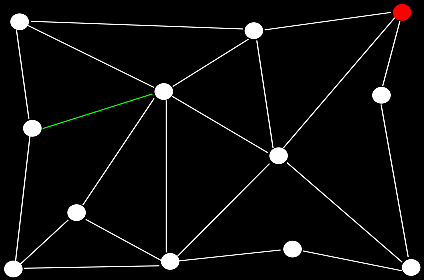

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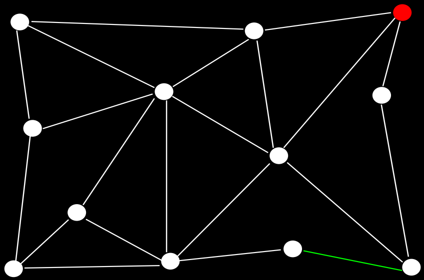

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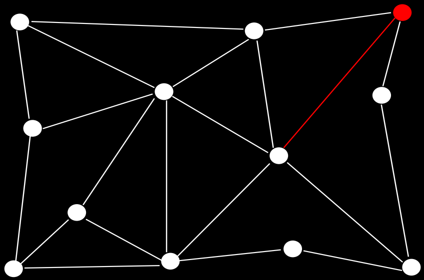

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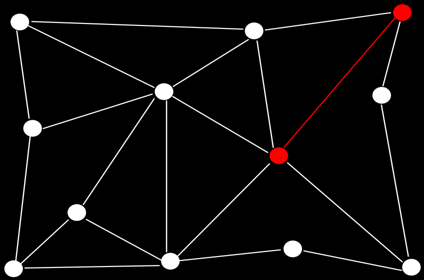

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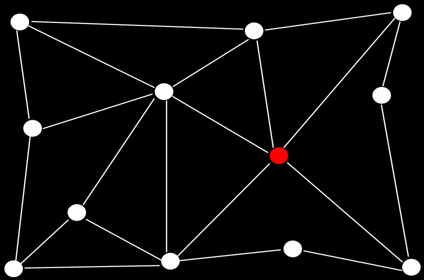

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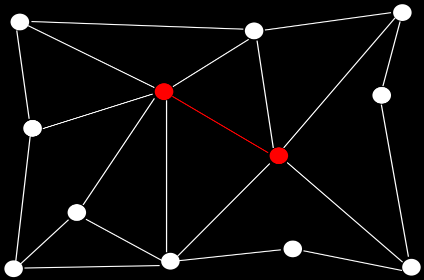

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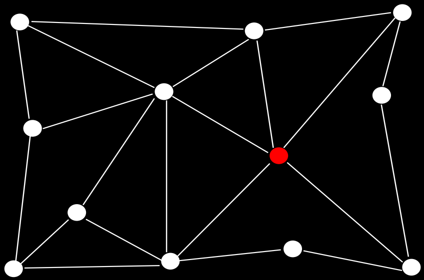

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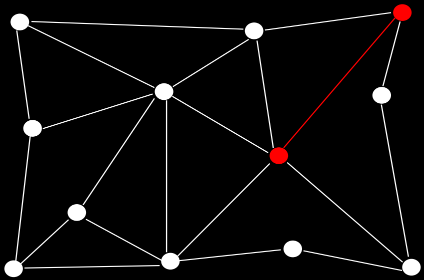

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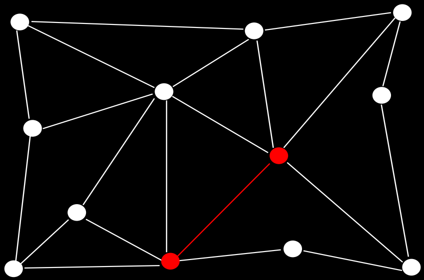

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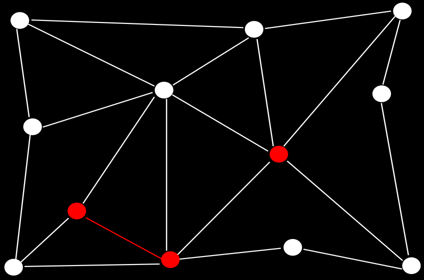
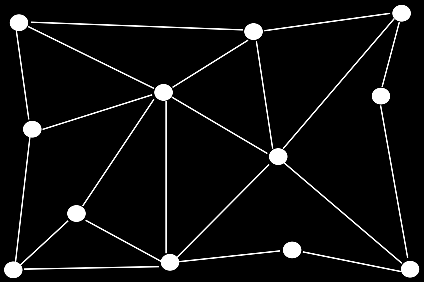


- S=susceptible I=infected
- $X_v(t) \in \{S, I\}$
- a random link gets activated at t (meeting)
- in finite (very very large) T $x_v(T) = S$ for all v
- metastable state with nontrivial fraction of $x_v = S$

More dynamics



- majority voter
- threshold models
- Moran process: evolution
- games
- dynamic flows
- ...

Analysis techniques

- Perron-Frobenius theory of positive systems
- monotone dynamical systems
- fast-mixing
- coupling
- duality
- martingale arguments
- mean-field limits
- branching process approximations
- density evolution
- ...

Next meetings: conference room M-building 1st floor

- Wed, October 2, 10:15-12:00: lecture
- Tue, October 8, 13:15-15:00: exercise
- Wed, October 9, 10:15-12:00: lecture
- Tue, October 15, 13:15-15:00: exercise
- Wed, October 16, 10:15-12:00: lecture
- Tue, October 22, 13:15-15:00: exercise
- Wed, October 23, 10:15-12:00: lecture
- Tue, October 29, 13:15-15:00: exercise
- Wed, October 30, 10:15-12:00: lecture
- Tue, November 5, 13:15-15:00: exercise
- Wed, November 6, 10:15-12:00: lecture
- Tue, November 12, 13:15-15:00: exercise
- Wed, November 13, 10:15-12:00: lecture
- Tue, November 19, 13:15-15:00: exercise
- Wed, November 20, 10:15-12:00: lecture
- Tue, November 26, 13:15-15:00: exercise
- Wed, November 27, 10:15-12:00: lecture