Exercise session 7

H_∞ Optimization of Coprime Factors. H_∞ Loop Shaping. ν-Gap Metric

Reading Assignment

Read [Zhou] Ch. 16, 17.2-3. Optional reading:

- McFarlane D. and K. Glover, "Robust Controller Design Using Normalized Coprime Factor Plant Descriptions", Lecture Notes in Control, 138, Springer-Verlag, 1990.

Exercises

E7.1 [Zhou] 16.3

E7.2 [Zhou] 16.5 (+16.6)

E7.3 [Zhou] 16.8

E7.4 Consider a simplified model of a satellite with two highly flexible solar arrays (Salehi, 10th IFAC Symposium, 1985)

$$\begin{aligned}
\dot{x} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & -2\zeta\omega \end{pmatrix} x + \begin{pmatrix} 0 \\ 1.7319e{-}05 \\ 0 \\ 3.7859e{-}04 \end{pmatrix} u + \begin{pmatrix} 0 \\ 1.7319e{-}05 \\ 0 \\ 3.7859e{-}04 \end{pmatrix} v, \\
y &= \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix} x.
\end{aligned}$$

where $\omega = 1.539 rad/sec$ is the frequency of the flexure mode and $\zeta = 0.003$ is the flexural damping ratio. Here u is the control torque (Nm), v is a constant disturbance torque (Nm) and y is the roll angle measurement (rad).

Required performance specification: y must stay within $0.04^\circ \approx 0.0007 rad$ pointing accuracy due to $0.3 Nm$ step torque disturbances.

1. Using [Zhou,16.12] prove the following bound

$$|(I - PK)^{-1} P| \leq \frac{\gamma}{|W|}$$

where $W = W_1 W_2$ is a single shaping function (for SISO plants).

2. Show that the performance specification is met for the constant shape function $W = 2000$.

3. Find the controller by H_∞ loop shaping procedure. Draw the Bode plots of the nominal P, shaped P_s and achieved WPK_∞ open loops. Compare.

4. Due to small steady-state error ($\approx 0.018°$) introduce another shaping function
$$W = \frac{10000(s+0.4)}{s}.$$
Show that the performance specification is met and the full steady-state disturbance rejection is guaranteed.

5. Repeat Item (c) for the new W.

E7.5 Calculate $\delta_\nu(P_0, P_z)$ and $\delta_\nu(P_0, P_\eta)$ for plants
$$P_0 = \frac{10(s^2+1)}{s^2(s^2+2)}, \quad P_z = \frac{10(s^2+1.1)}{s^2(s^2+2)}, \quad P_\eta = \frac{10(s^2+1)}{s^2(s^2+1.8)}$$
and conclude about a sensitivity of δ_ν to the variation in the imaginary axis zeros and poles. Explain the result from the Riemann sphere interpretation of δ_ν.

Hand-In problems:

H7.1 [Zhou] 16.9. Plot the Bode diagram of the specified P_s and the achieved $WK_\infty P$ loop shape.

H7.2 Let a nominal plant be given by $P = \frac{s-1}{s(s+2)}$ and we select $W = k$, a constant, as the shaping function. Thus $P_s = \frac{k(s-1)}{s(s+2)}$.

1. Calculate $b_{opt}(P_s)$ and $\delta_\nu(P, P_s)$ for 4 different choices of $W = k$
$$k = [.1\ 1\ 5\ 10].$$
What k's give better loop shape for controller design?

2. For all 4 choices of W: Find the optimal controller K_∞. Plot the Bode diagram of the specified loop shape P_s and the achieved loop shape $WK_\infty P$.

3. Explain the result from performance limitation point of view.