Exercise session 6

H_∞ Optimization Problem. Frequency Domain Approach. Algebraic Riccati Equations. State Space Solution.

Reading Assignment

Read [Zhou] Ch. 12,14. Optional reading:

- Frequency Domain [Francis]
- ARE and State Space [Zhou,Doyle,Glover]
- Doyle J., Glover K., Khargonekar P., Francis B., *State Space Solution to Standard H^2 and H^∞ Control Problem*, IEEE Trans. on AC **34** (1989) 831–847.

Exercises

E6.1 [Zhou] 12.1

E6.2 [Zhou] 14.5 (and 13.4)

E6.3 [Zhou] 14.6

E6.4 [Zhou] 14.7

E6.5 [Zhou] 14.11

E6.6 Exercise 5.4 continued: (c–v) Using μ-synthesis technique design a stabilizing controller K which guarantees RP taking into account the structure of Δ.

Hand-In problems:

H6.1 [Zhou] 13.2 and 14.3. Plot the Bode diagram of the closed-loop transfer function for both problems and compare them. Conclusion?

H6.2 For each of the following systems

$$\begin{aligned} G_1 &= \frac{1}{(s+1)^3}, \\ G_2 &= \frac{1}{(s^2+0.14s+1)(s+1)}, \\ G_3 &= \frac{1}{(s^2-0.14s+1)(s+1)} \end{aligned}$$

design a H_∞ controller that minimizes the cost function

$$\left\| \begin{pmatrix} W_s S \\ W_u K S \end{pmatrix} \right\|_\infty$$

where

$$W_s = \frac{s/M + \omega_B}{s + \omega_B A}$$

with $M = 2$, $\omega_B = 5$ and $A = 0.01$. The constant weight W_u should be adjusted to make the cost function smaller than one.

For each case, draw the following plots:

- Sensitivity to show that you meet specs.
- The Nyquist/Nichols plot to check stability.
- Bode graph of G_i, K and $L = G_i K$. Clearly label each curve and identify the gain/phase cross-over point.
- The root locus obtained by using a controller αK where $\alpha \in [0, 2]$.
- The step response of the closed-loop system.