Exercise session 3

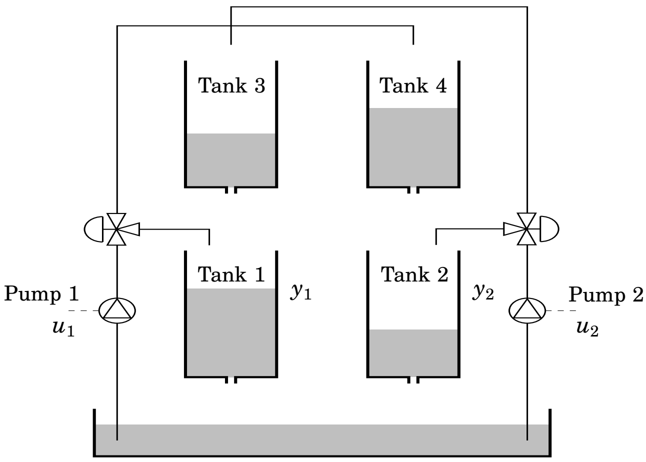
Reading Assignment

Read [Zhou] Ch. 6.
Optional reading: [Doyle,Francis,Tannenbaum] Ch. 6, 7.2 and Glad/Ljung, *Control Theory — Multivariable and Nonlinear Methods*, Taylor & Francis, 2000.

E3.1 The four tank process has the transfer matrix

$$P(s) = \begin{pmatrix} \dfrac{\gamma_1}{1+s} & \dfrac{(1-\gamma_2)}{(1+s)(1+s)} \\ \\ \dfrac{(1-\gamma_1)}{(1+s)(1+s)} & \dfrac{\gamma_2}{1+s} \end{pmatrix}$$

where γ_1 and γ_2 correspond to the valve settings.



(a) Compute the zeros of the transfer matrix as functions of γ_1 and γ_2.

(b) What restrictions do they impose on the output sensitivity function?

(c) Define some first order weighting matrix $W(s)$ for which the specification

$$\sup_{\omega} \|W(i\omega)[I + P(i\omega)C(i\omega)]^{-1}\| \leq 1$$

is impossible to satisfy?

E3.2 Zhou 6.9

E3.3 Zhou 6.10

E3.4 The specifications

$$\sup_{\omega} |W_S(i\omega)S(i\omega)| \leq 1 \qquad\qquad \sup_{\omega} |W_T(i\omega)T(i\omega)| \leq 1$$

can be used to make sure that the sensitivity is small for frequencies $<< a$ and measurement noise is rejected for frequencies $>> b$.

(a) Show that the two specifications are incompatible if

$$|W_S(s)| = |W_T(s)| > 2 \qquad (*)$$

for some right half plane s. (Hint: Use that $S + T = 1$.)

(b) What relation does the constraint $(*)$ impose on a and b in case that

$$W_S(s) = \left(\frac{s+a}{s}\right)^n \qquad\qquad W_T(s) = \left(\frac{s+b}{b}\right)^n \qquad ?$$

Hint: Note that $|W_S(s)| = |W_T(s)|$ for $s = \sqrt{ab}$.
Compute actual numbers for the case $n = 3$.

E3.5 Let $P(s) = 4(s-2)/(s+1)^2$. Suppose that K is an internally stabilizing controller such that $\|S\|_\infty = 1.5$. Give a positive lower bound for

$$\sup_{0 \leq \omega \leq 0.1} |S(j\omega)|.$$

Hand-In problems:

1. [Zhou] 6.4
2. [Zhou] 6.6