Control Systems with Actuator Saturation:

Anti-windup Mechanism and Its Activation

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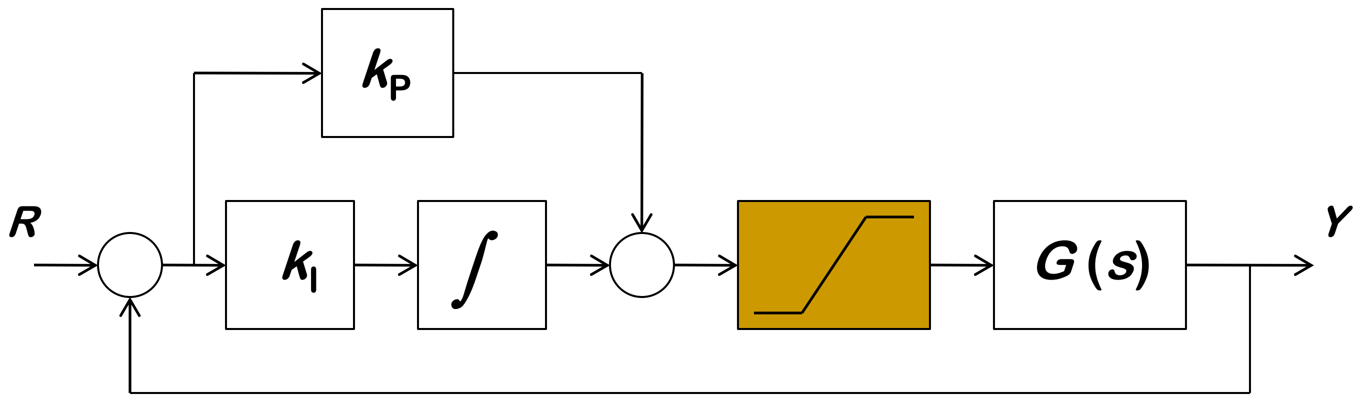
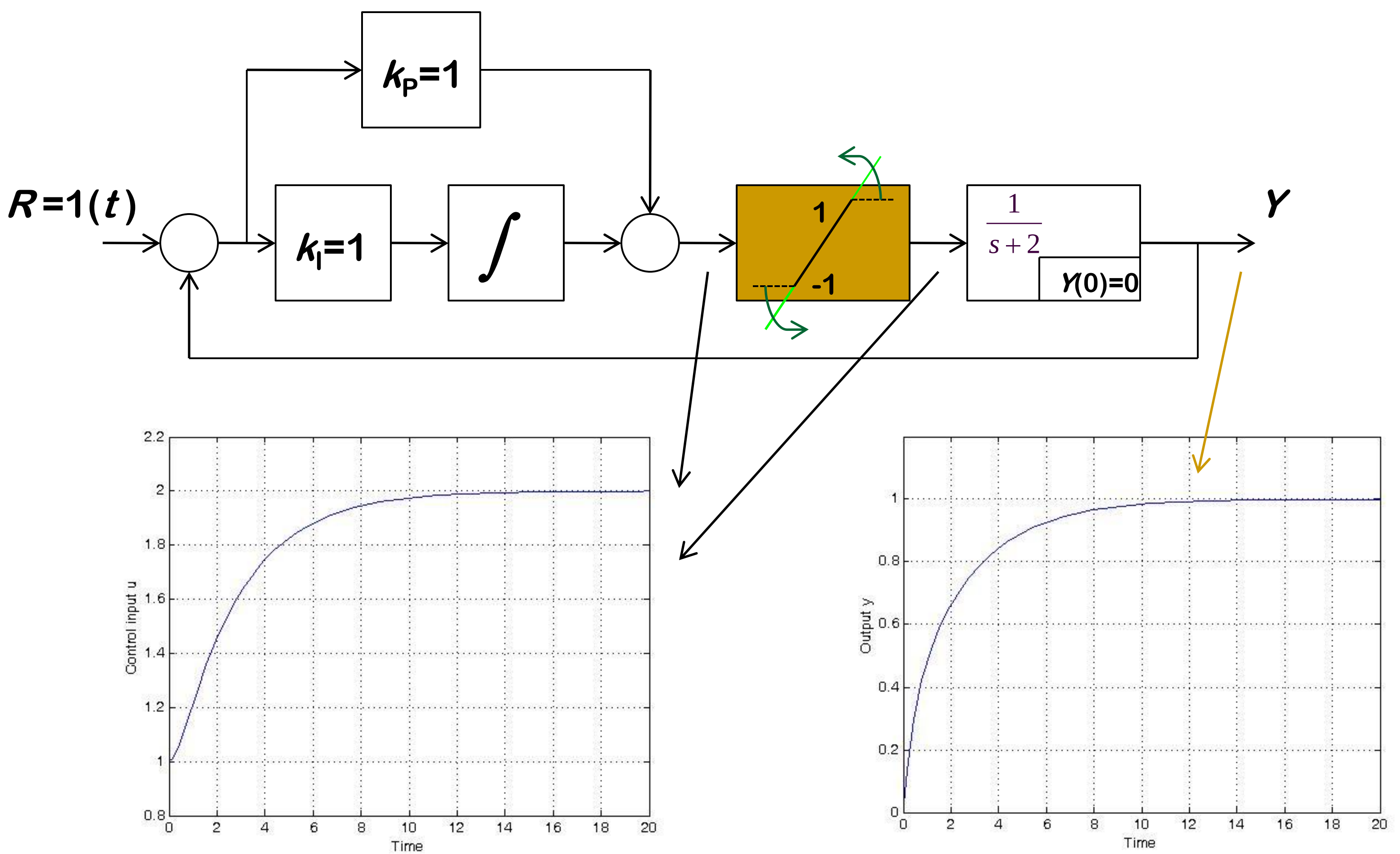
1 Actuator Saturation and Integrator Windup



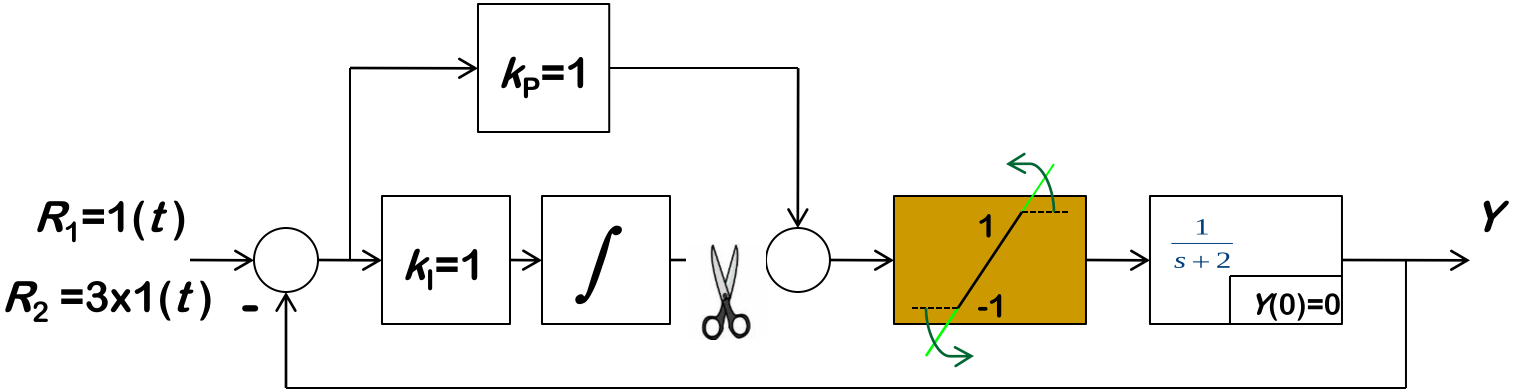
- Actuator saturation is a common phenomenon
- Integrators are commonly present in a controller

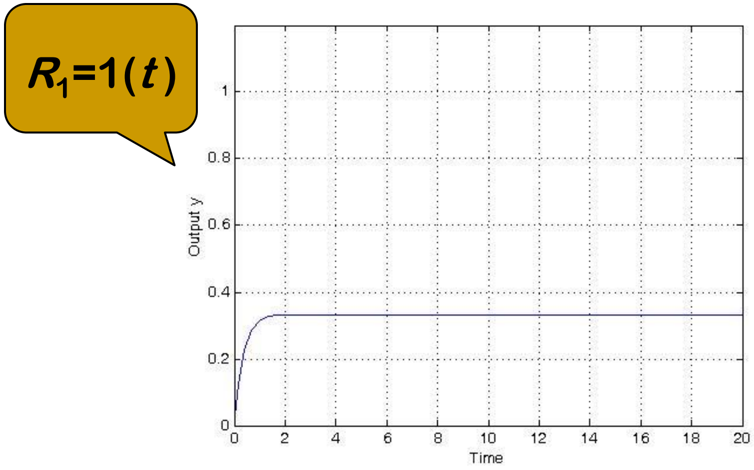
1 Actuator Saturation and Integrator Windup

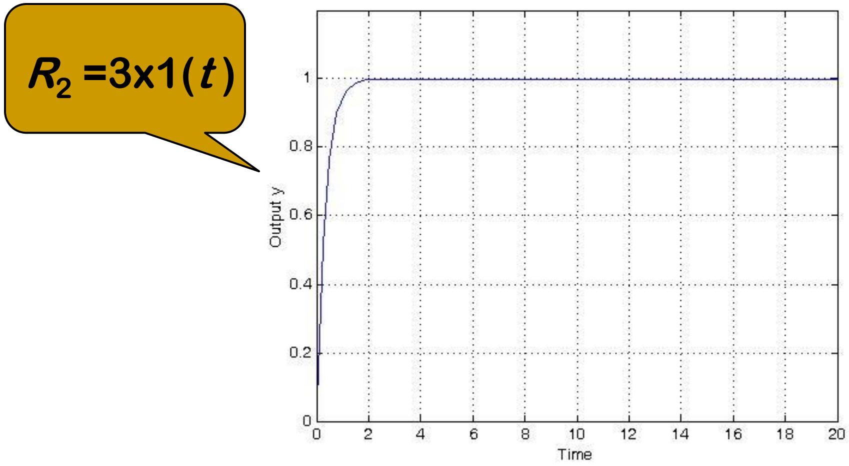


1 Actuator Saturation and Integrator Windup

➢ **Role of integrator: elimination of steady state error = reset of reference**







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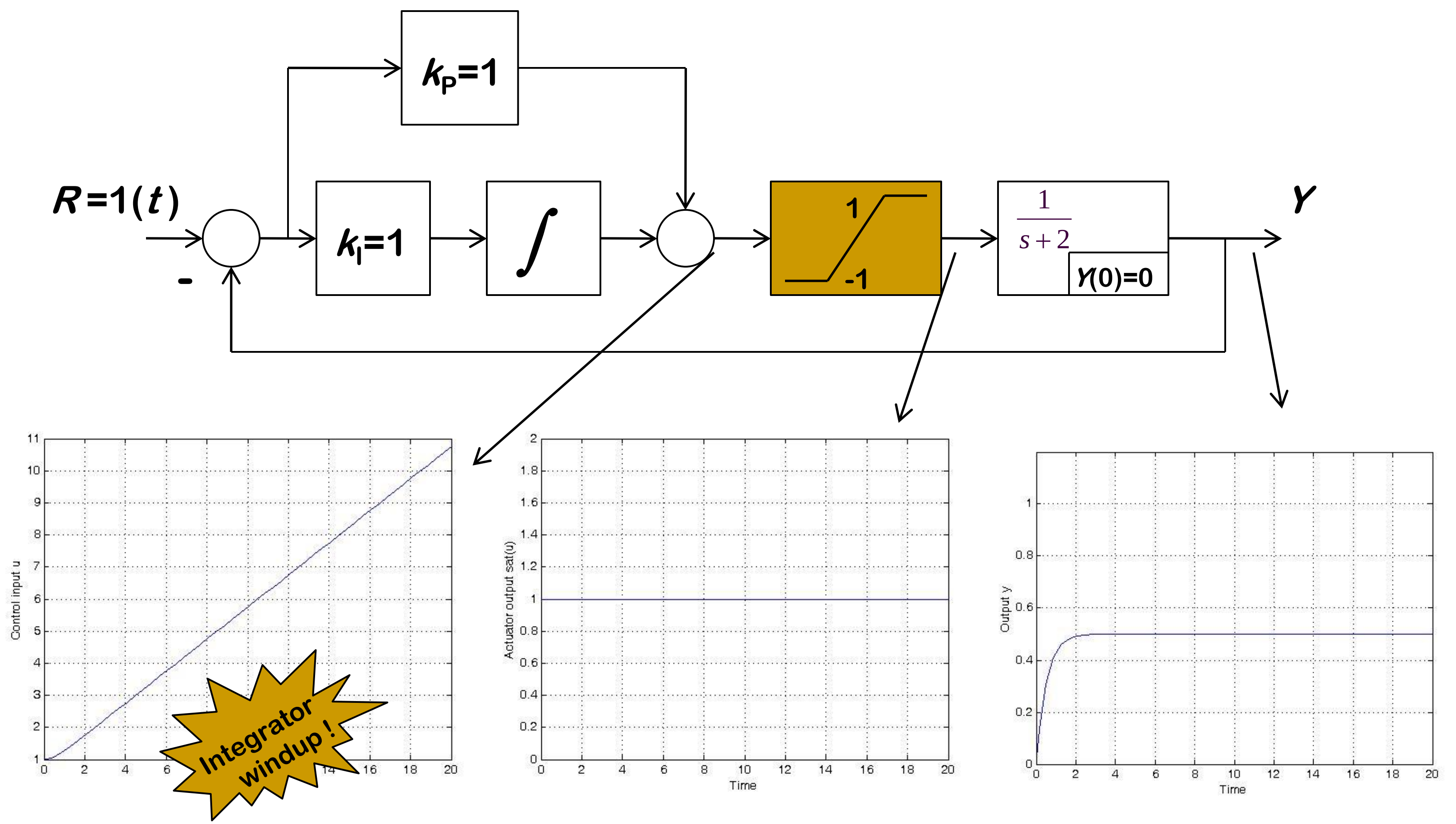
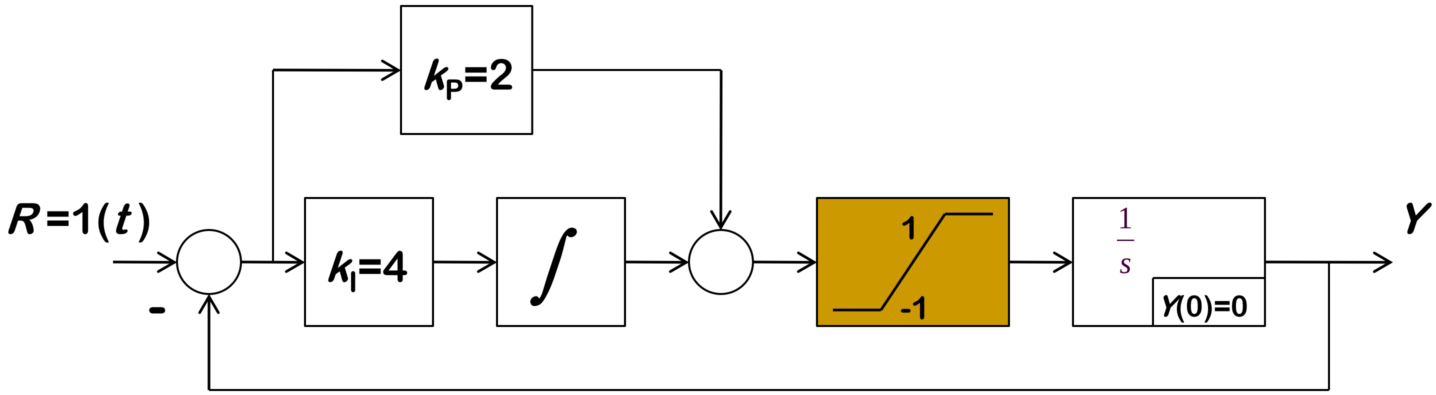
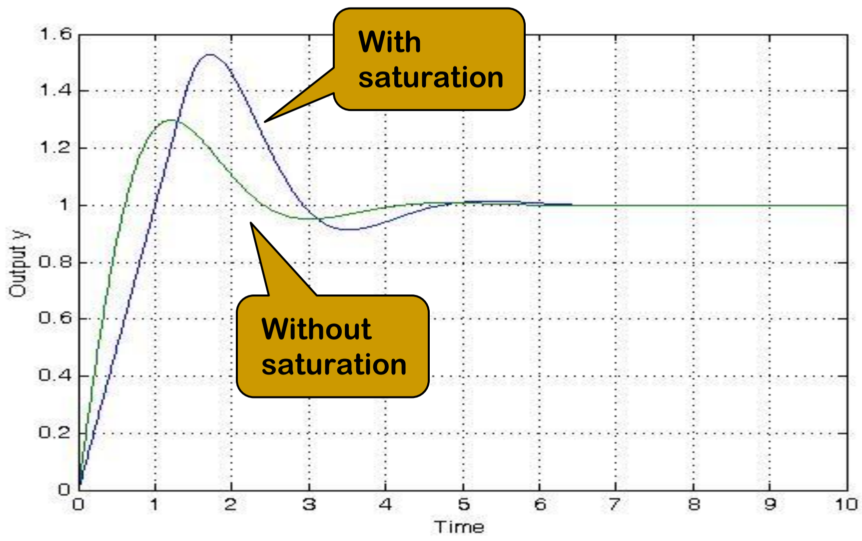
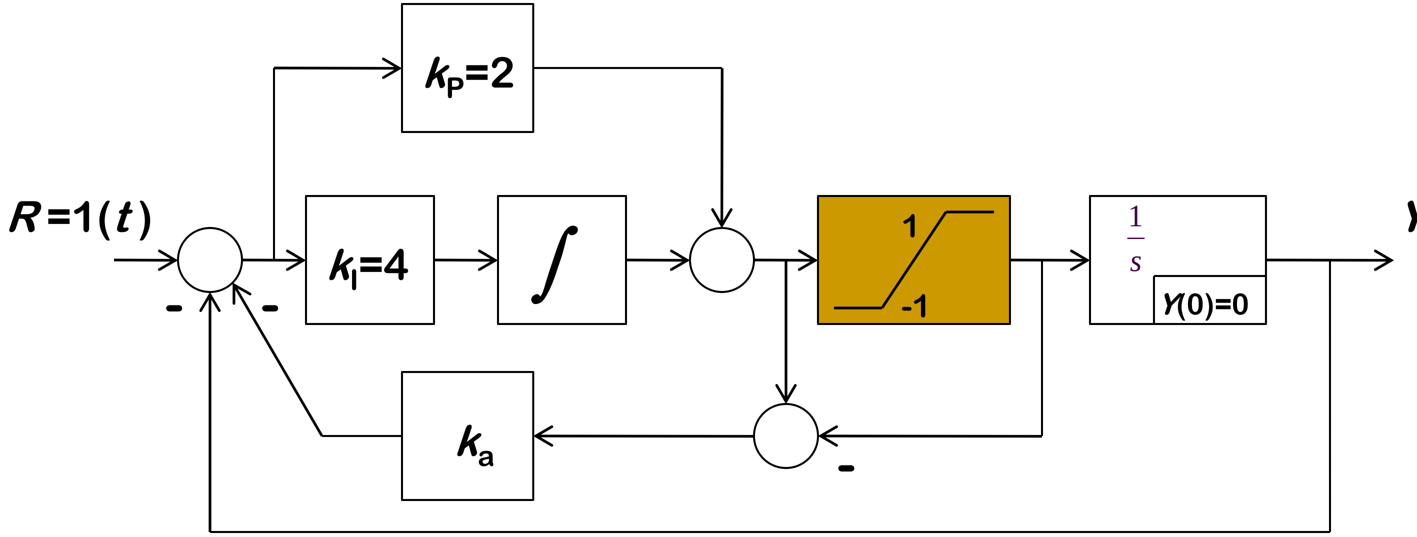


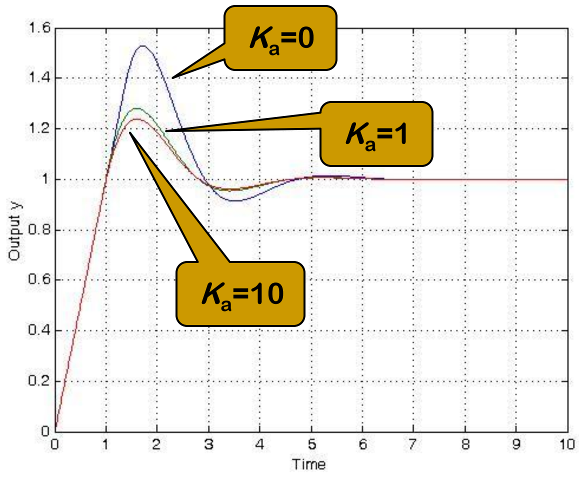
1 Actuator Saturation and Integrator Windup

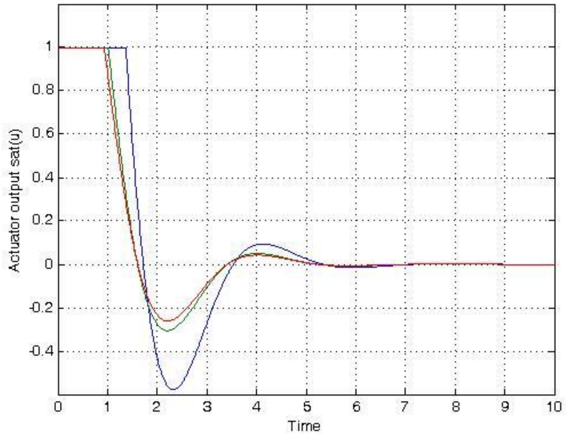




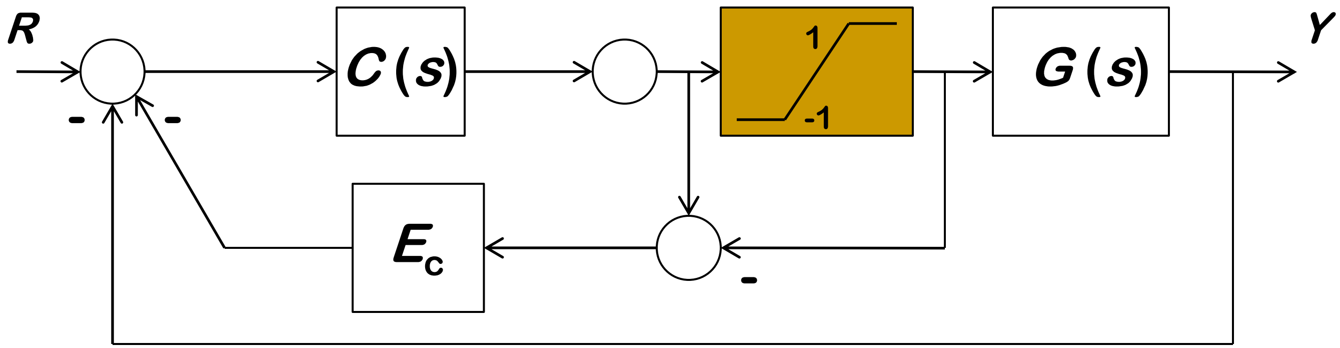
Actuator Saturation and Integrator Windup







2 Anti-windup Mechanism and Its Activation

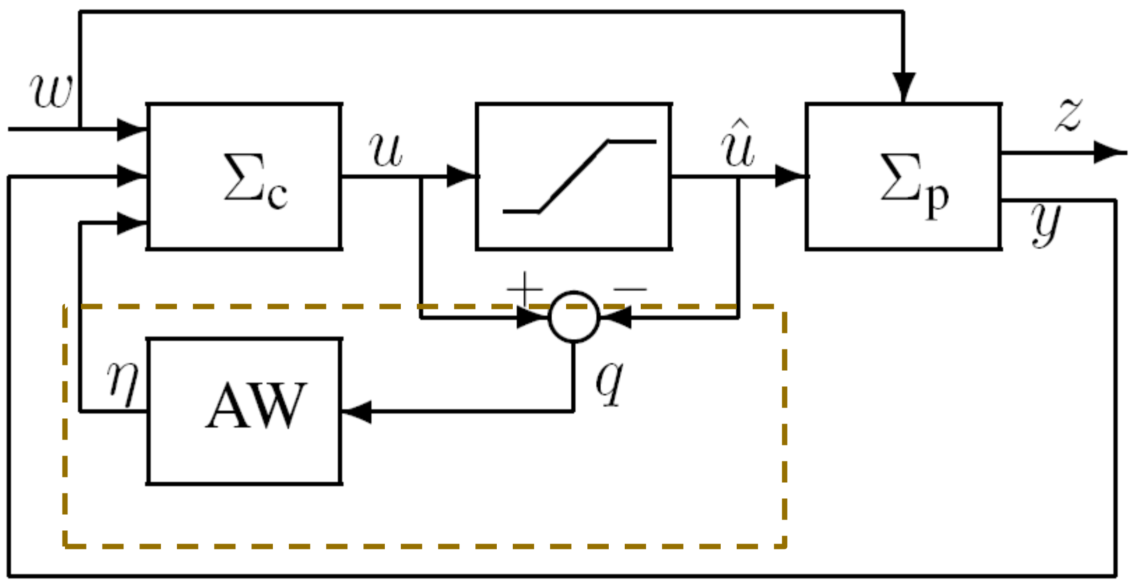


Systematic design of E_c that guarantees

- a large size of stability region (domain of attraction)
- good closed-loop performances such as a small L_2 gain from the disturbance to the output

2 Anti-windup Mechanism and Its Activation

The traditional approach to anti-windup design is to activate the anti-windup mechanism as soon as the saturation occurs.

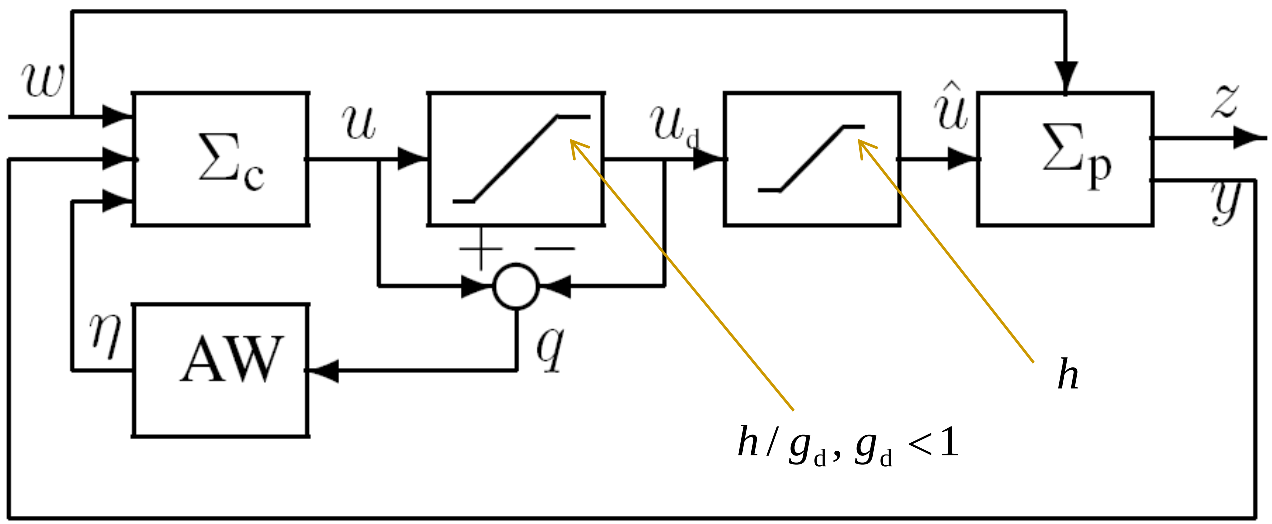


Immediate Activation

2 Anti-windup Mechanism and Its Activation

A Recent Innovation [Sajjadi-Kia & Jabbari, CDC'09]:

To delay the activation of the anti-windup mechanism until the degree of saturation reaches to a certain level.



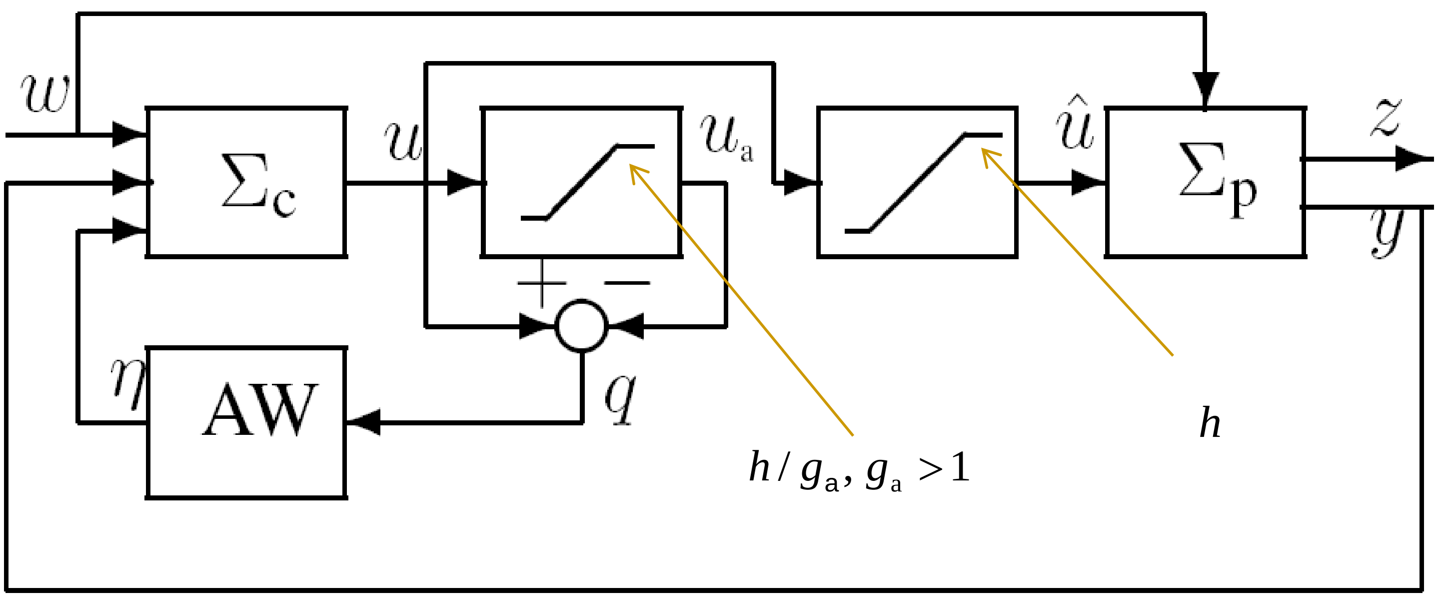
Delayed Activation

Result: **Improved transient performance.**

2 Anti-windup Mechanism and Its Activation

Our Proposal [Wu & Lin, CDC'10]:

To activate the anti-windup mechanism in anticipation of the occurrence of saturation.



Anticipatory Activation

Problem:

To examine the effectiveness of anticipatory activation of anti-windup mechanism in

1. L_2 gain anti-windup design

2. anti-windup design for large region of stability

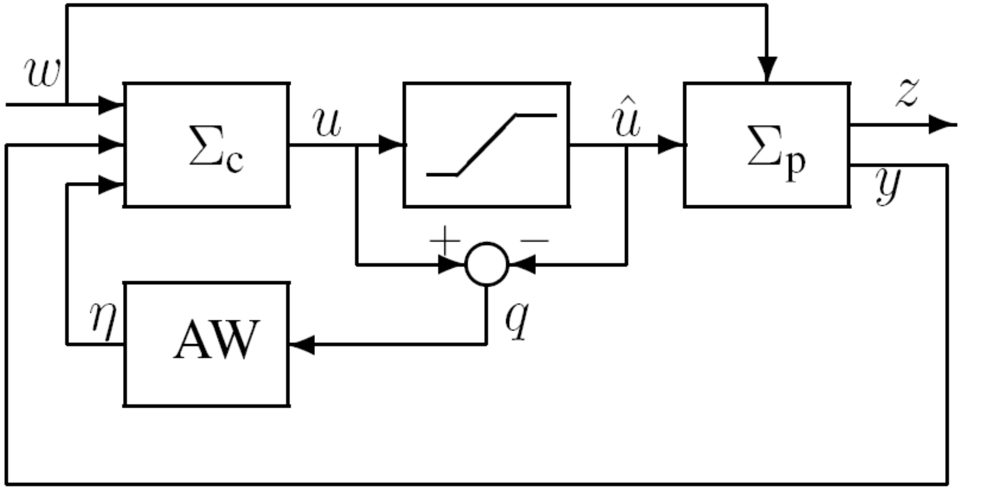
3 L_2 Gain Anti-windup Design

Consider the following system

$$\begin{cases} \dot{x}_{\mathrm{p}} &= A_{\mathrm{p}} x_{\mathrm{p}} + B_{\mathrm{w}} w + B_2 \mathrm{sat}_h(u) \\ y &= C_2 x_{\mathrm{p}} + D_{21} w + D_{22} \mathrm{sat}_h(u) \\ z &= C_1 x_{\mathrm{p}} + D_{11} w + D_{12} \mathrm{sat}_h(u) \end{cases}$$

Let a linear dynamic controller be given in the form of

$$\begin{cases} \dot{x}_{\mathrm{c}} &= A_{\mathrm{c}} x_{\mathrm{c}} + B_{\mathrm{cy}} y + B_{\mathrm{cw}} w \\ u &= C_{\mathrm{c}} x_{\mathrm{p}} + D_{\mathrm{cy}} y \end{cases}$$



3 L_2 Gain Anti-windup Design

Under the immediate activation scheme, a static anti-windup design is to add to the controller a correction term proportional to $q = u - \mathrm{sat}_h(u)$ as soon as saturation occurs:

$$\begin{cases} \dot{x}_c &= A_c x_c + B_{cy} y + B_{cw} w - \eta_1 \\ u &= C_c x_p + D_{cy} y - \eta_2 \end{cases}$$

where,

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = -\Lambda q = -\begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} q$$

This leads to the following closed-loop system:

$$\begin{cases} \dot{x} &= Ax + B_w w + (B_q - B_\eta \Lambda) W^{-1} \tilde{q} \\ z &= C_z x + D_{zw} w + (D_{zq} - D_{z\eta} \Lambda) W^{-1} \tilde{q} \\ \tilde{u} &= W C_u x + W D_{uw} w + W (D_{uq} - D_{u\eta} \Lambda) W^{-1} \tilde{q} \\ \tilde{q} &= W^{-1} \Delta W C_u \tilde{u} \end{cases}$$

where,

$$q = u - \mathrm{sat}(u) = \Delta u,\ x = \begin{bmatrix} x_p \\ x_c \end{bmatrix}$$

3 L_2 Gain Anti-windup Design

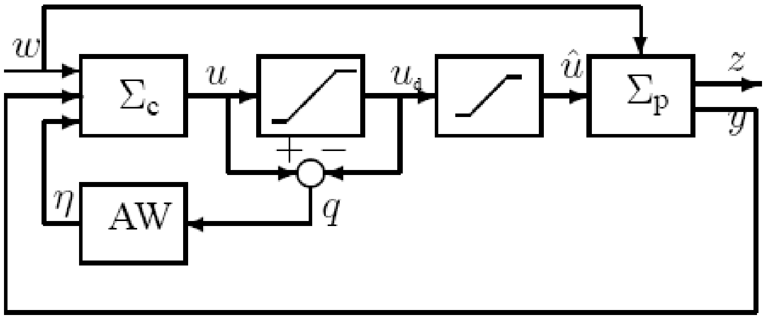
Theorem [Sajjadi-Kia & Jabbari, CDC'09]: The closed-loop system is stable and the L_2 gain from w to z is less than γ if there exist a diagonal matrix $M > 0$, a matrix $Q > 0$ and a vector X such that

$$\begin{bmatrix} AQ+QA^{\mathrm{T}} & * & * & * & * \\ MB_{\mathrm{q}}^{\mathrm{T}} - X^{\mathrm{T}}B_{\eta}^{\mathrm{T}} & -M & * & 0 & * \\ C_{\mathrm{u}}Q & D_{\mathrm{uq}}M - D_{\mathrm{u}\eta}X & -M & * & 0 \\ B_{\mathrm{w}}^{\mathrm{T}} & 0 & D_{\mathrm{uw}}^{\mathrm{T}} & -\gamma I & * \\ C_{\mathrm{z}}Q & D_{\mathrm{zq}}M - D_{\mathrm{z}\eta}X & 0 & D_{\mathrm{zw}} & -\gamma I \end{bmatrix} < 0.$$

If this LMI is feasible, then the anti-windup gain can be obtained as $\Lambda = XM^{-1}$.

L_2 Gain Anti-windup Design

Delayed activation scheme:



Theorem [Sajjadi-Kia & Jabbari, '09]:

The closed-loop system is stable with an L_2 gain from w to z less than γ if there exist a diagonal matrix $M > 0$, a matrix $Q > 0$ and a vector X such that

$$\begin{bmatrix} \Omega_1(g) & * & * & * & * \\ B_{\mathrm{w}}^{\mathrm{T}}(g) & -\gamma I & 0 & * & * \\ \Omega_2(g) & 0 & -M & * & 0 \\ C_{\mathrm{z}}(g)Q & D_{zw}(g) & \Omega_3(g) & -\gamma I & 0 \\ C_{\mathrm{u}}Q & D_{uw} & \Omega_4(g) & 0 & -M \end{bmatrix} < 0, g = g_{\mathrm{d}}, \quad \begin{bmatrix} A(g)Q + QA^{\mathrm{T}}(g) & * & * \\ B_{\mathrm{w}}(g) & -\gamma I & * \\ C_{\mathrm{z}}(g)Q & D_{zw}(g) & -\gamma I \end{bmatrix} < 0, g = 1$$

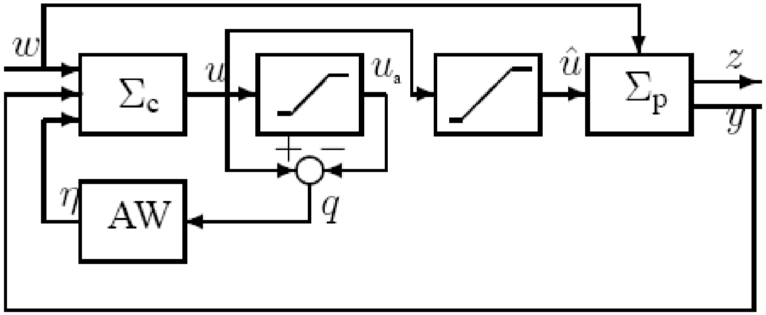
where $\Omega_1(g) = A(g)Q + QA^{\mathrm{T}}(g),\ \Omega_2(g) = MB_{\mathrm{q}}^{\mathrm{T}}(g) - X^{\mathrm{T}} B_{\eta}^{\mathrm{T}}(g),$

$\Omega_3(g) = D_{zq}(g)M - D_{z\eta}(g)X,\ \Omega_4(g) = D_{uq}(g)M - D_{u\eta}(g)X$

The anti-windup gain can then be obtained as: $\Lambda = XM^{-1}$.

3 L_2 Gain Anti-windup Design

Anticipatory activation scheme:



Theorem: The closed-loop system is stable and the L_2 gain from w to z is less than γ if there exist a diagonal matrix $M>0$, a matrix $Q>0$ and a vector χ such that

$$\begin{bmatrix} A(g)Q+QA^{\mathrm{T}}(g) & * & * \\ B_{\mathrm{w}}(g) & -\gamma I & * \\ C_{\mathrm{z}}(g)Q & D_{\mathrm{zw}}(g) & -\gamma I \end{bmatrix} < 0, g=1, \quad \begin{bmatrix} \Omega_1(g) & * & * & * & * \\ B_{\mathrm{w}}^{\mathrm{T}}(g) & -\gamma I & 0 & * & * \\ \Omega_2(g) & 0 & -M & * & 0 \\ C_{\mathrm{z}}(g)Q & D_{\mathrm{zw}}(g) & \Omega_3(g) & -\gamma I & 0 \\ C_{\mathrm{u}}Q & D_{\mathrm{uw}} & \Omega_4(g) & 0 & -M \end{bmatrix} < 0, g=\{1, g_{\mathrm{a}}\}.$$

The anti-windup gain can then be obtained as: $\Lambda = XM^{-1}$.

3 L_2 Gain Anti-windup Design

Simulation Results

Consider plant and controller with $h = 1$,

$$\left[\begin{array}{c|c|c} A_p & B_2 & B_1 \\ \hline C_2 & D_{22} & D_{21} \\ \hline C_1 & D_{11} & D_{12} \end{array}\right] = \left[\begin{array}{ccc|c|c} -10.6 & -6.09 & -0.9 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline -1 & -11 & -30 & 0 & 0 \\ \hline -1 & -11 & -30 & -1 & 0 \end{array}\right], \quad \left[\begin{array}{c|c|c} A_c & B_{cy} & B_{cw} \\ \hline D_c & D_{cy} & D_{cw} \end{array}\right] = \left[\begin{array}{cc|c|c} -80 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ \hline 20.15 & 1600 & 80 & -80 \end{array}\right]$$

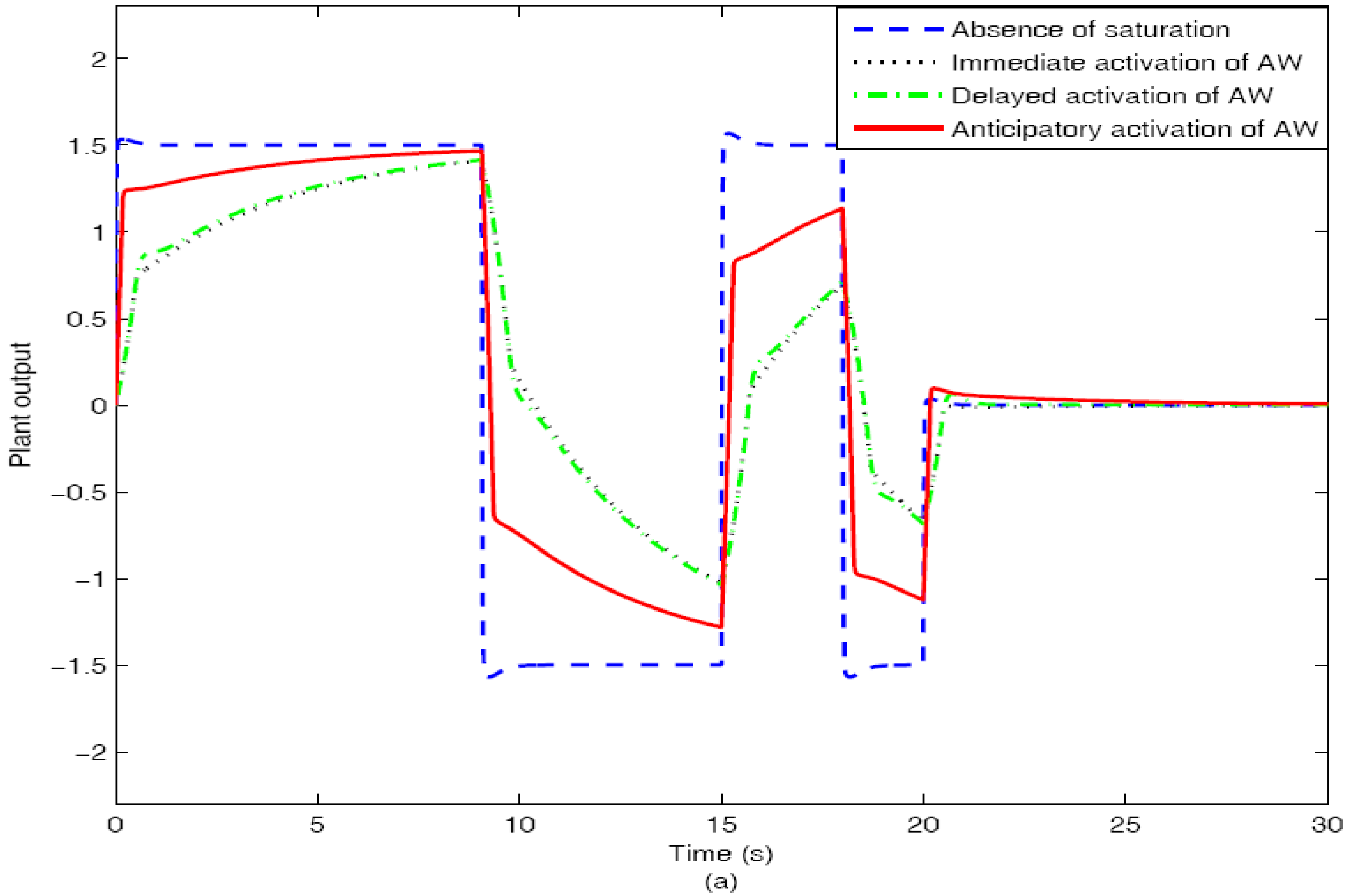
and select $g_d = 0.1700$, $g_a = 1.002$.

Immediate activation scheme: $\gamma = 85.7800$

Delayed activation scheme: $\gamma = 87.4124$

Anticipatory activation scheme: $\gamma = 86.5200$

3 L_2 Gain Anti-windup Design



Transient performance under different activation schemes

4 Anti-windup Design for Large Region of Stability

Consider a linear system with actuator saturation

$$\begin{cases} \dot{x}_{\mathrm{p}} &= A_{\mathrm{p}} x_{\mathrm{p}} + B_{\mathrm{p}} \mathrm{sat}_h(u) \\ y &= C_{\mathrm{p}} x_{\mathrm{p}} \end{cases}$$

Assume that a linear dynamic compensator has been designed as

$$\begin{cases} \dot{x}_{\mathrm{c}} &= A_{\mathrm{c}} x_{\mathrm{c}} + B_{\mathrm{c}} y \\ u &= C_{\mathrm{c}} x_{\mathrm{c}} + D_{\mathrm{cy}} y \end{cases}$$

Objective: To examine how delayed and anticipatory activation of the anti-windup mechanism will affect the size of the domain of attraction of the resulting closed-loop system as compared to the immediate activation scheme.

4 Anti-windup Design for Large Region of Stability

With the immediate activation scheme:

The controller law with anti-windup compensation is given by

$$\begin{cases} \dot{x}_c = A_c x_c + B_c y + E_c(\mathrm{sat}_h(u) - u) \\ u = C_c x_c + D_{cy} y \end{cases}$$

The closed-loop system can be written as

$$\dot{x} = (A - BF)x + B\mathrm{sat}_h(Fx)$$

where

$$x = \begin{bmatrix} x \\ x_c \end{bmatrix}, A = \begin{bmatrix} A_p + B_p D_c C & B_p C_c \\ B_p C_c & A_c \end{bmatrix},$$

$$B = \begin{bmatrix} B_p \\ E_c \end{bmatrix}, F = \begin{bmatrix} D_c C_p & C_c \end{bmatrix}.$$

4 Anti-windup Design for Large Region of Stability

The design of E_c can be formulated into the following optimization problem with BMI constraints

$$\max_{P>0, E_c, H} \quad \alpha$$

s.t. a) $\alpha X_R \subset \varepsilon(P)$

b) Binlinear matrix nequalities

c) $\varepsilon(P) \subset L(H)$

where $X_R = \mathrm{co}\{x_1, x_2, \cdots, x_l\}$, for some a priori given points $x_i \in R^n$, is referred to as a shape reference set.

An iterative LMI based algorithm was developed in [Cao, Lin & Ward, TAC '02] to solve the above BMI optimization problem.

Similar algorithms can be developed under the framework of delayed activation and anticipatory activation.

Simulation examples

Consider a stable plant

$$\begin{cases} \dot{x}_1 & = & -0.1x_1 + 0.5\text{sat}(u_1) + 0.4\text{sat}(u_2) \\ \dot{x}_2 & = & -0.1x_2 + 0.4\text{sat}(u_1) + 0.3\text{sat}(u_2) \end{cases}$$

or an unstable plant with the first equation replaced by

$$\dot{x}_1 = 0.1x_1 - 0.2x_2 + 0.5\text{sat}(u_1) + 0.4\text{sat}(u_2)$$

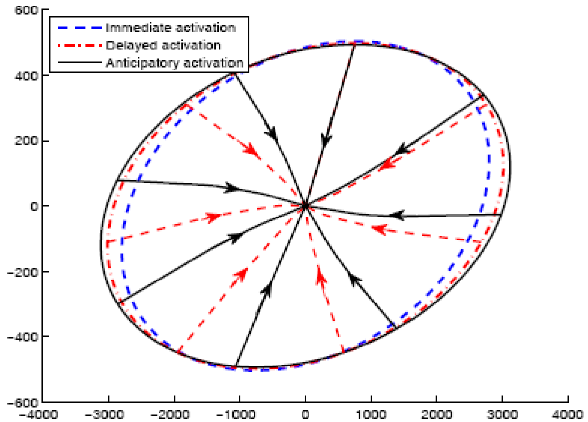
Let a PI controller be given by

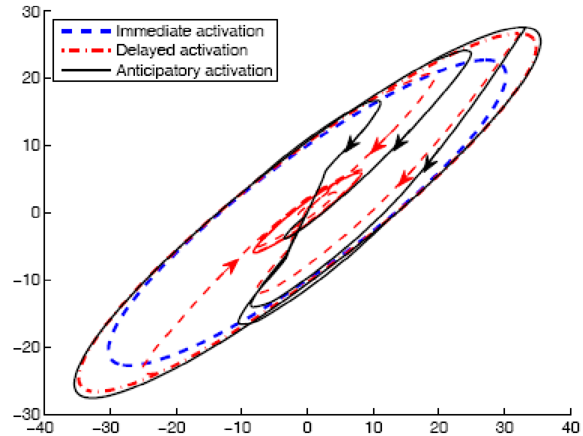
$$\begin{cases} \dot{x}_{\text{c1}} = -x_1, \quad \dot{x}_{\text{c1}} = -x_2 \\ u_1 = -10x_1 + x_{\text{c1}}, \quad u_2 = 10x_1 - x_{\text{c1}} \end{cases}$$

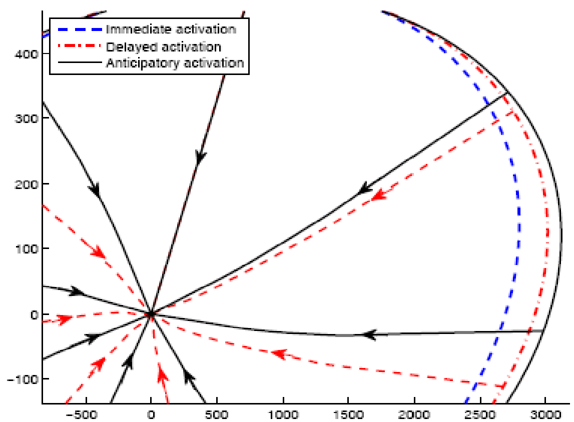
Let $g_{\text{d}} = 0.9470$, **and** $g_{\text{a}} = 1.0030$, **we can obtain**

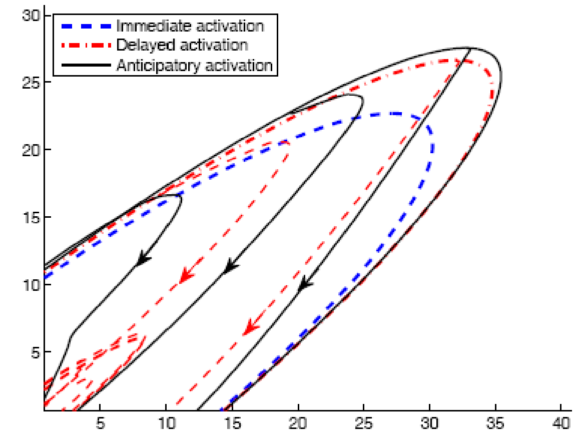
$$E_{\text{c delayed}} = \begin{bmatrix} 74.9597 & 50.7074 \\ 56.9891 & 52.1316 \end{bmatrix}, \quad E_{\text{c anticpatory}} = \begin{bmatrix} 84.5677 & 53.8271 \\ 64.2234 & 54.5939 \end{bmatrix}.$$

4 Anti-windup Design for Large Region of Stability





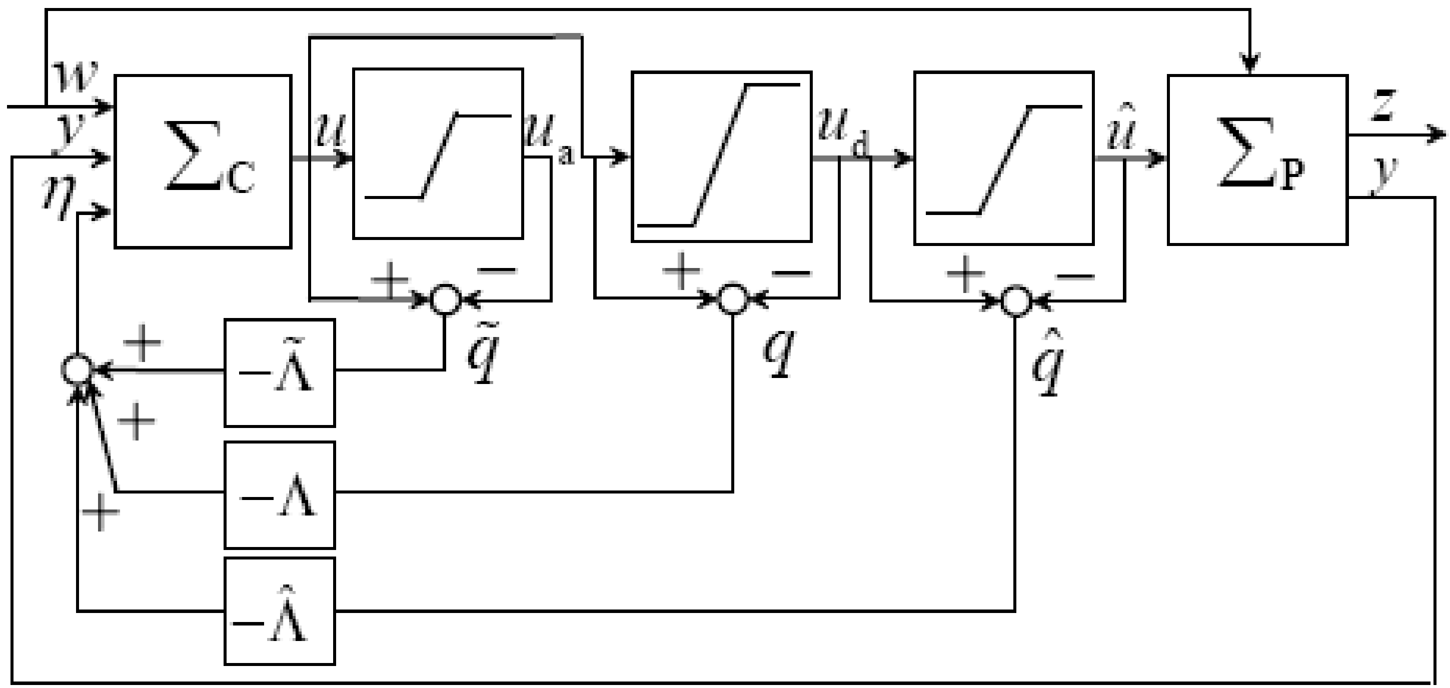




Stable plant **Unstable plant**

5 Multiple Activations of Anti-windup Mechanism

Wu & Lin, CCC'11:



Multiple Anti-windup Loops for Multiple Activations

5 Multiple Activations of Anti-windup Mechanism

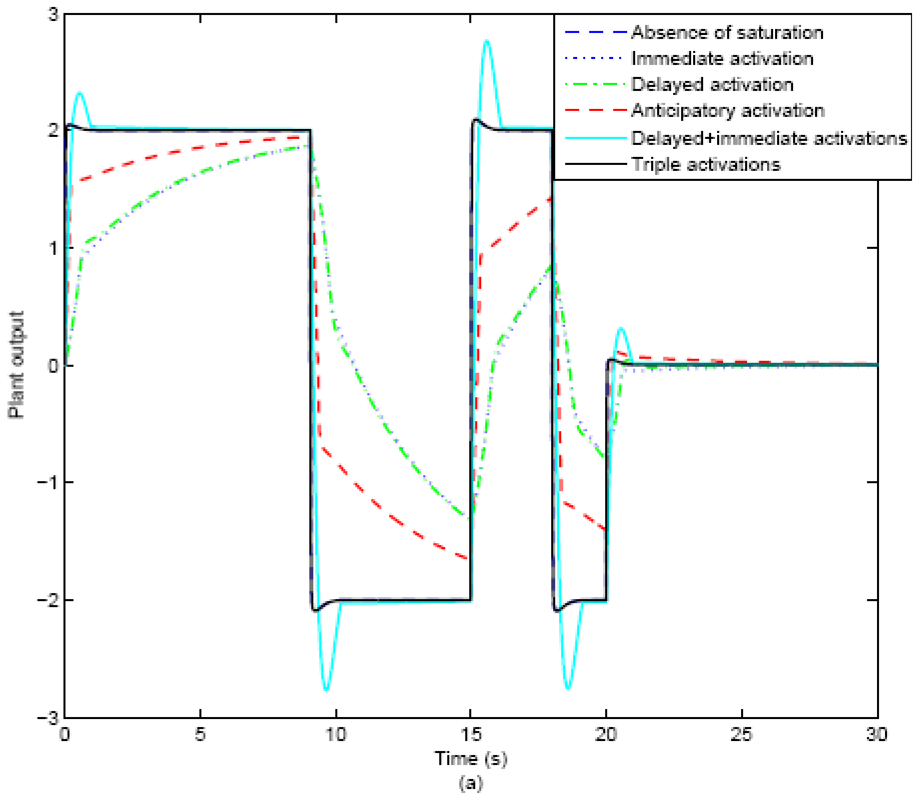
L_2 gain anti-windup design:

$$\left[\begin{array}{c|c|c} A_p & B_2 & B_1 \\ \hline C_2 & D_{22} & D_{21} \\ \hline C_1 & D_{11} & D_{12} \end{array}\right] = \left[\begin{array}{ccc|c|c} -10.6 & -6.09 & -0.9 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline -1 & -11 & -30 & 0 & 0 \\ \hline -1 & -11 & -30 & -1 & 0 \end{array}\right], \quad \left[\begin{array}{c|c|c} A_c & B_{cy} & B_{cw} \\ \hline D_c & D_{cy} & D_{cw} \end{array}\right] = \left[\begin{array}{cc|c|c} -80 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ \hline 20.15 & 1600 & 80 & -80 \end{array}\right]$$

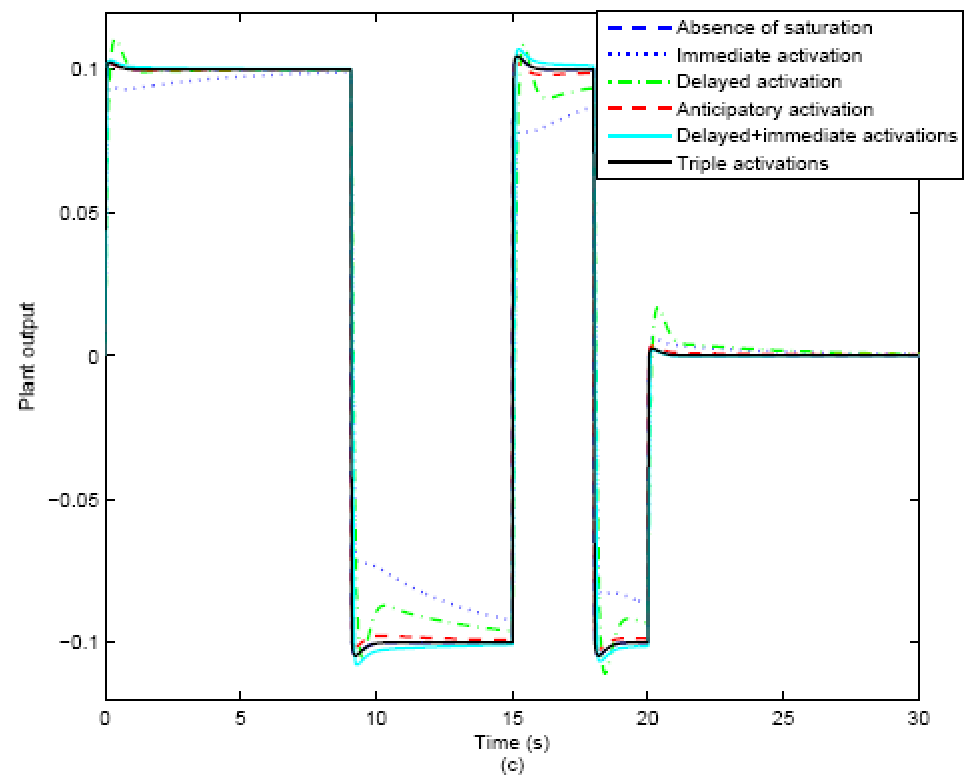
$$\Lambda = \begin{bmatrix} 2.7266\times10^{-4} \\ 3.4085\times10^{-6} \\ 2.3022\times10^{-2} \end{bmatrix}, \quad \tilde{\Lambda} = \begin{bmatrix} 3.0093\times10^{-4} \\ 3.7616e\times10^{-6} \\ 9.8555\times10^{-1} \end{bmatrix}, \quad \hat{\Lambda} = \begin{bmatrix} 4.5475\times10^{1} \\ 4.5475\times10^{1} \\ 7.5044\times10^{1} \end{bmatrix}$$

3

L_2 Gain Anti-windup Design: Multiple Case



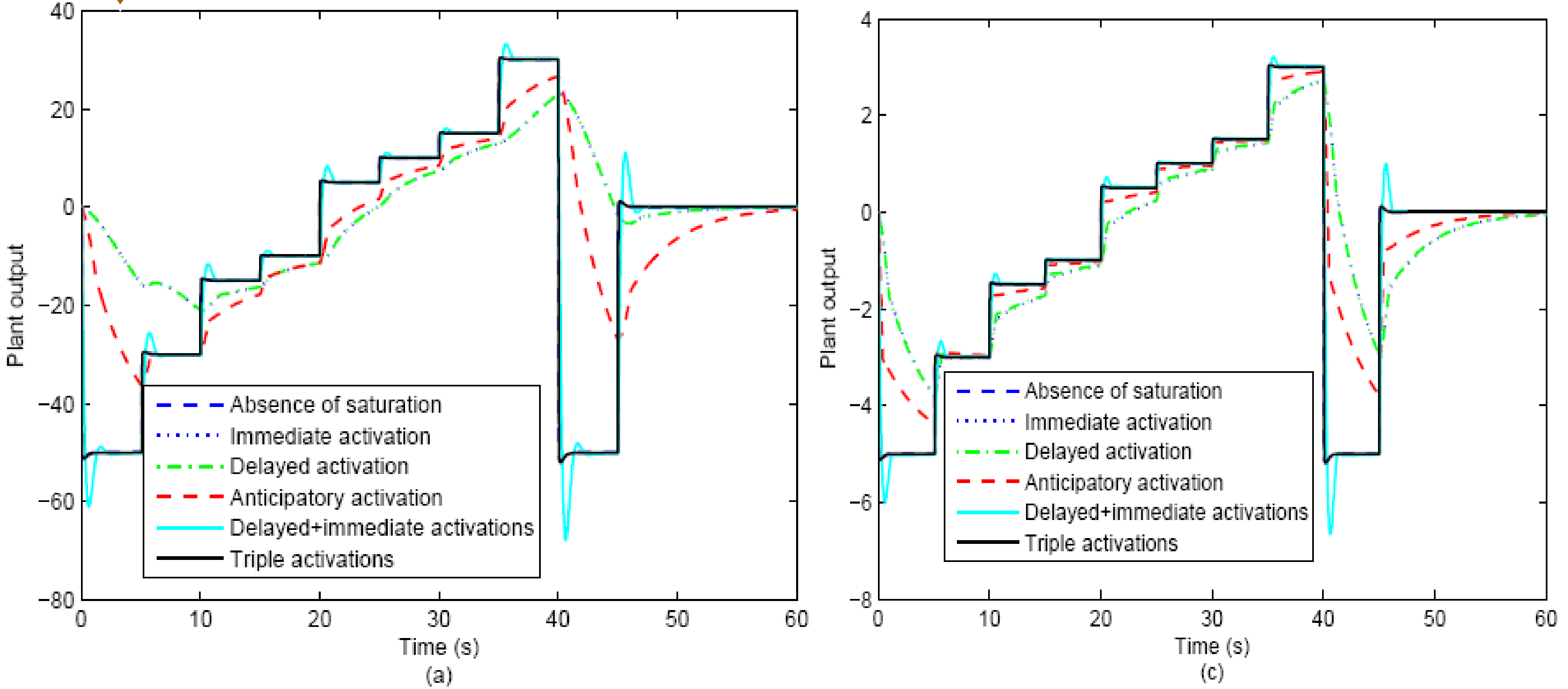
large input



small input

Transient performance under different activation schemes

3 L_2 Gain Anti-windup Design: Multiple Case



large input **small input**

Transient performance under different activation schemes

6 Questions and Comments