Adaptive Perturbation Method for Global Stabilization of Minimally Rigid Formations in the Plane

Yu-Ping Tian, Qin Wang

School of Automation, Southeast University

Nanjing, China

Outline

Formation in nature

Formation control:

Control a group of objects so that they move in a formation.

Why formation?

- save energy (birds, fish),
- accomplish tasks that are difficult for individuals, such as group difence, hunting, transportation, etc.

Application of formation control

- Movement control of unmanned aerial vehicles (UAVs)
- Location of mobile nodes of sensor networks
- Robot soccer
- ……

Approaches to Formation Control

A1

Relative position control: the desired formation is specified by the desired relative position vectors.
Feature: a consensus problem,
global stabilization by linear feedback control law

A2

Relative distance control: the desired formation is specified by the desired relative distances.
Feature: nonlinear feedback control law

- **multiple equilibria problem**

Approaches to Formation Control

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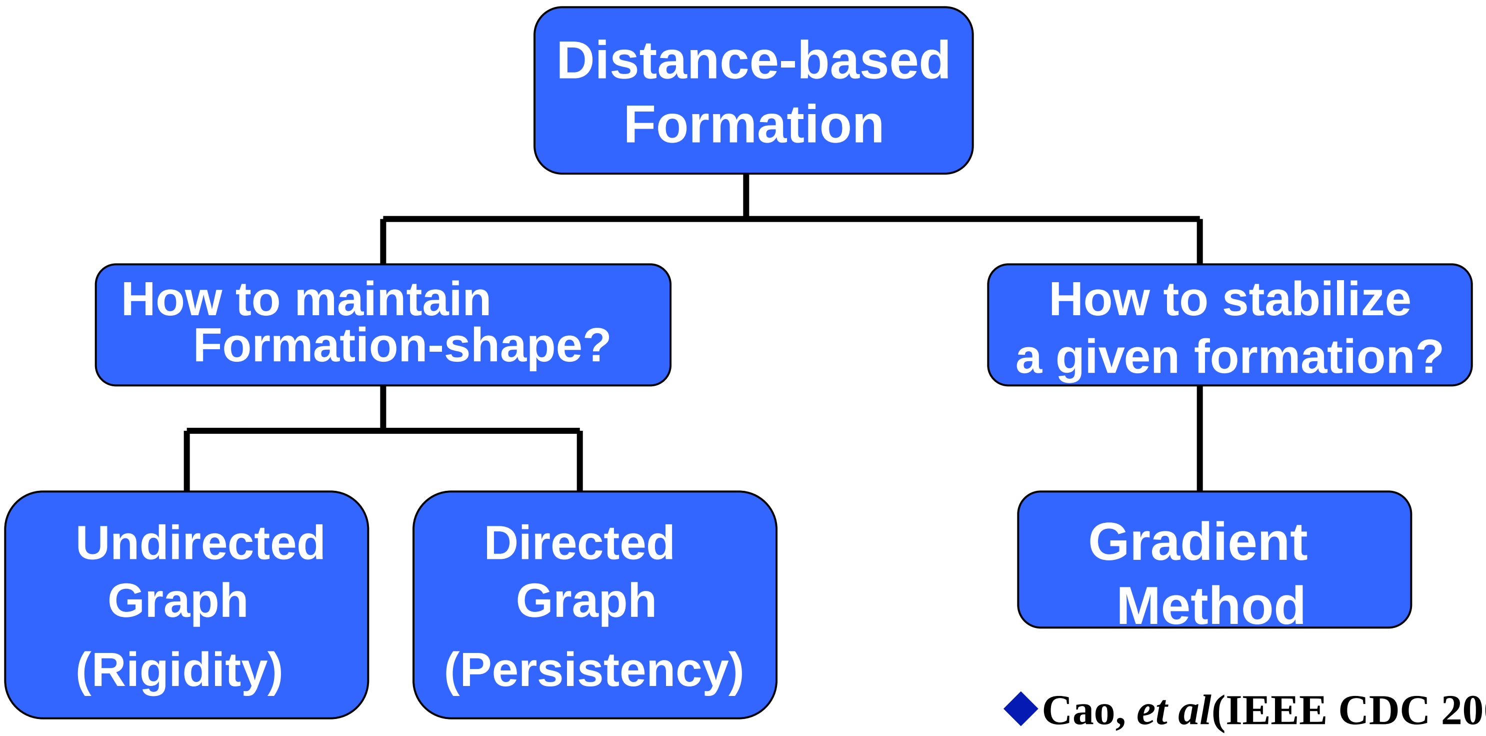
Relative distance control: the desired formation is specified by the desired relative distances.
Feature: nonlinear feedback control law

- **multiple equilibria problem**

Single equilibrium problem, global stabilization

Brief review of Relative Distance Control approach



Laman (1970)

see, e.g., Yu, Anderson (2007, Automatica)

- **Cao, *et al*(IEEE CDC 2007)**
- **Krick, *et al*(IJC, 2009)**
- **Dimarogonas & Johansson(ACC, 2009)**
- **Bai He(ACC, 2010)**

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1. Introduction
2. Problem Statement and Preliminaries
3. Gradient Control Law and Local Stability
4. Global Stabilizer with Adaptive Perturbation
5. Simulations

System model: agent

- **Mass-point model of agent** (1)

$$\dot{r}_i = v_i, \quad i = 1, \cdots, n$$

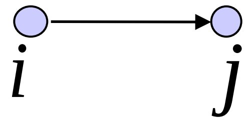
where $r_i = [x_i, y_i]^T \in R^2$ and $v_i = [v_{xi}, v_{yi}]^T \in R^2$ are the position and velocity, respectively, of agent i .

v_i is the control input of agent i .

System model: Interconnection graph and formation

◈ Information architecture

$$G=(V,E), V=\{1,2,\cdots,n\}, E\subseteq V\times V$$



an Edge means

i) j is a neighbor of i , $j\in N_i$

ii) i measures $r_{ij}=r_i-r_j$

iii) i knows its desired distance to j must be $d_{ij}, d_{ij}=d_{ji}$

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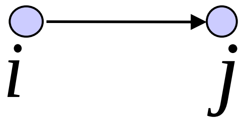

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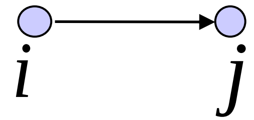

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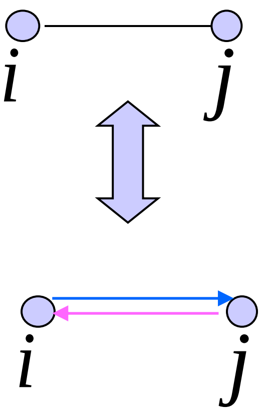

System model: Interconnection graph and formation

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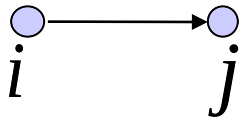

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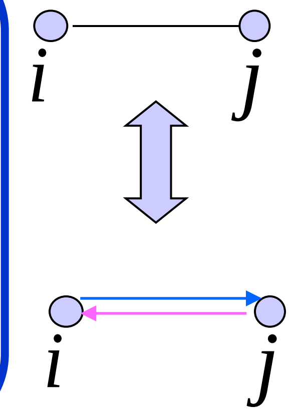


Let $D=\{d_{ij}: i\in V, j\in N_i\}$ be a set of desired distances associated with G.

Then, the framework $\{G,D\}$ represents a desired formation.

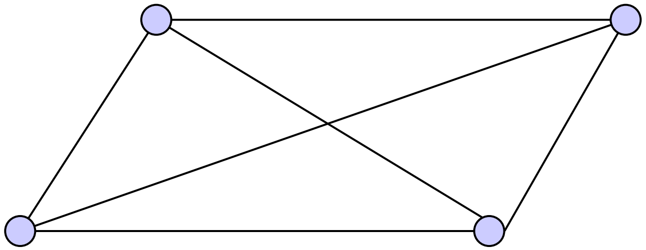
Control objective

The formation control problem is to design a undirected control law based on the relative positions measurement, such that for any initial position, the multi-gents systems can achieve the globally asymptotically stable rigid formation, and no collision happens between any two adjacent agents.

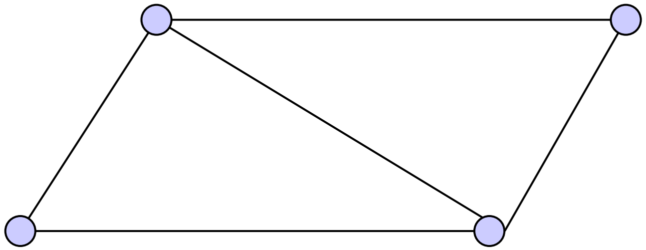
Rigid Graph

The formation $\{G, D\}$ is said to be ***rigid*** if provided that all distance constraints required by $\{G, D\}$ are satisfied during a continuous displacement, all inter-vertex distances remain constant.

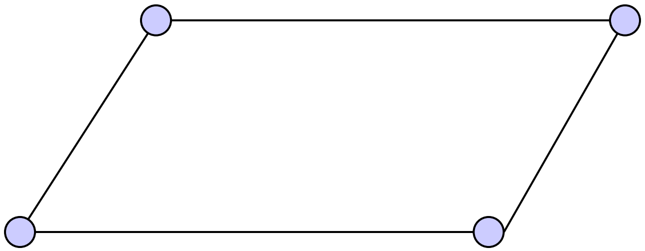
The formation $\{G, D\}$ is ***minimally rigid*** if it is rigid and if there is no rigid graph having the same vertices but less edges.



(a) Rigid graph



(b) Minimally rigid graph



(c) non-rigid graph

Henneberg sequence

A Henneberg sequence is a sequence of graphs $G_2, G_3, \cdots, G_{|V|}$ with $G_2 = K_2$ being the complete graph on two vertices, and where each graph $G_i (i \geq 3)$ can be obtained from G_{i-1} by either ***a vertex addition operation*** or ***an edge splitting operation***.

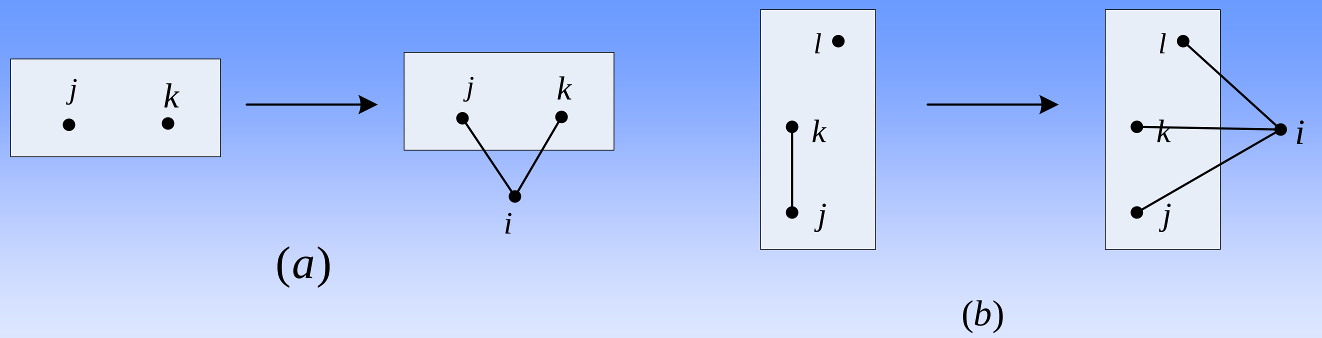


Figure 1: Representation of vertex addition operation in (a) and of the edge plitting operation in (b)

Rigid graph

The following result gives a constructive method to form a minimally rigid graph.

Lemma 1 [Hendrickx]: Every minimally rigid graph on more than one vertex can be obtained as the result of a Henneberg sequence. Moreover, all the graphs of such a sequence are minimally rigid.

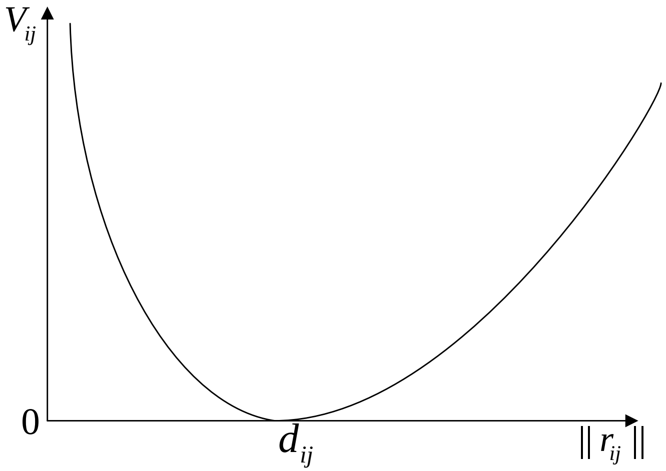
The following result gives quantitative relationship between the numbers of edges and vertices of a minimally rigid graph.

Lemma 2 [Hendrickx]: A graph $G(V, E)$ with $|V| > 1$ is minimally rigid if and only if $|E|=2|V|-3$ and for all non-empty subset E' of E, there holds $|E'|\leq 2|V(E')|-3$, where $2|V(E')|$ is the set of vertices incident to E'.

Potential Functions

Potential Function : **Dimarogonas&Johansson(CDC, 2008)**

$$V_{ij} = \frac{(\alpha_{ij} - d_{ij}^2)^2}{\alpha_{ij}} \qquad (2)$$



where $\alpha_{ij} = \| r_{ij} \|^2$

We also define $\rho_{ij} = \frac{\partial V_{il}(\alpha_{il})}{\partial \alpha_{il}} = \frac{\alpha_{ij}^2 - d_{ij}^4}{\alpha_{ij}^2}$

Note that $\rho_{ij} = \rho_{ji}$, for all $i, j \in N, i \neq j$

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Previous Results: Gradient Method

Dynamics

$$\dot{r}_i = v_i \quad r_i, v_i \in R^2, i = 1, 2, \cdots, n$$

Gradient control law

$$v_i = -\nabla_{r_i} V_i = -\sum_{j \in N_i} \nabla_{r_i} V_{ij}(\| r_{ij} \|), \ j \in N_i$$

Previous Results: Gradient Method

Cao, *et al* (IEEE CDC 2007)

Consider 3-agent directed formation control.

The negative gradient control law can drag the system from any non-collinear initially positions to a desired triangular formation.

Krick, *et al* (IJC, 2009)

Consider *n*-agent undirected formation control.

By using the center manifold theory, it is proved that the desired formation is locally asymptotically stable under the gradient control law.

Dimarogonas &Johansson (ACC, 2009)

Under the negative gradient control laws multi-agents system is globally stable with respect to the desired formation if and only if the formation graph is a tree.

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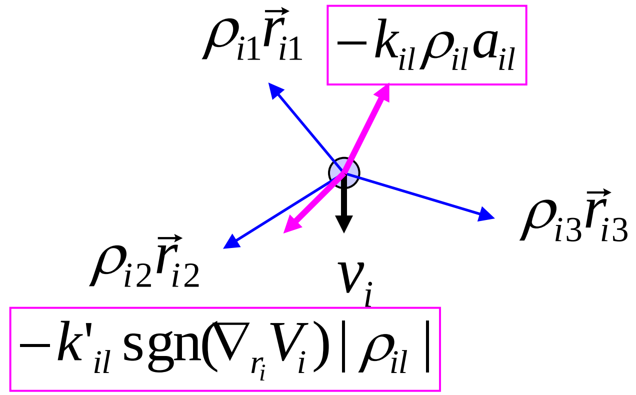
Global Stabilizer with Adaptive Perturbation

- Gradient control + adaptive perturbation

$$v_i = -\nabla_{r_i} V_i \quad -k_{il}\rho_{il}a_{il} - k'_{il}\,\mathrm{sgn}(\nabla_{r_i} V_i)\,|\rho_{il}|$$

$$= -\sum_{j\in N_i} \rho_{ij}\vec{r}_{ij} \quad \underline{-k_{il}\rho_{il}a_{il} - k'_{il}\,\mathrm{sgn}(\nabla_{r_i} V_i)\,|\rho_{il}|}$$



$-k_{il}\rho_{il}a_{il}$ Main term in perturbation, vanishing only if $\rho_{il} = 0$ i.e., $\|r_{il}\| = d_{il}$

(Strength is adaptively adjusted by ρ_{il}, orientation varies periodically with frequency ω_{il}.)

$-k'_{il}\,\mathrm{sgn}(\nabla_{r_i} V_i)\,|\rho_{il}|$ Stabilizing term in perturbation

(Strength is adaptively adjusted by $|\rho_{il}|$, orientation varies according to $\mathrm{sgn}(\nabla_{r_i} V_i)$.)

Remarks on the control law

1. Among *n* agents, only *n-2* agents are selected for adding perturbations.
2. To each selected agent, say agent *i*, only one neighbor (*l*) is chosen, according to which the perturbation combination

$$-k_{il}\rho_{il}a_{il} - k'_{il}\,\mathrm{sgn}(\nabla_{r_i} V_i)\,|\rho_{il}|$$

is determined.

How to arrange *n* - 2 agents to which the perturbation is added?

How to choose the neighbor *l* of the perturbed agent i by which the adaptive perturbation strength ρ_{il} is determined?

Decompose a minimally rigid graph as $G = G_t \bigcup G_c$, where G_t is a spanning tree of *G*, and G_c contains the remaining edges in *G*.

We can prove that if the distances of the edges in G_c can converge to the desired value under the perturbed gradient control, then the distances of the rest edges in *G* (the edges of the spanning tree) will automatically converge to the desired values under standard gradient control.

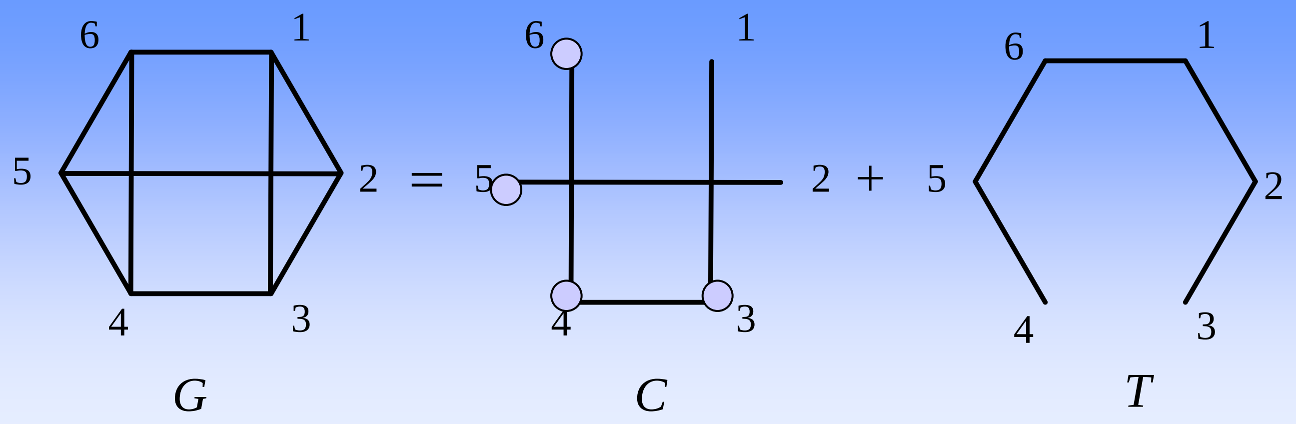
By Lemma 2, there are *n-2* edges in G_c .

Global Stabilizer with Adaptive Perturbation

Theorem 1

Suppose $G(V,E)$ is minimally rigid graph. There exists a decomposition $G=G_t \bigcup G_c$, such that the $n-2$ edges in sub-graph G_c can be looked after by $n-2$ distinct vertices.

Global Stabilizer with Adaptive Perturbation



The figure illustrates the decomposition of minimally rigid graph *G* with 6 vertices.

Stability Analysis

Theorem 2

Assume that the system (1) controlled by the control law (3)-(4), with the potential function V_{ij} as in (2). Then, the desired minimally rigid formation is globally asymptotically stable, the velocities of all the agents converge to zero, and no collision between any pair of adjacent agents happens during the motion.

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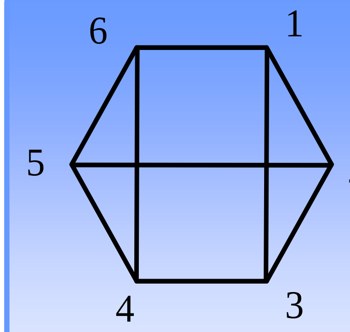
An example

Let the desired formation shape be given by Fig. 2.

The desired distance vector is given by

$$D=[d_{12},d_{23},d_{34},d_{45},d_{56},d_{61},d_{13},d_{46},d_{25}]^T$$

$$d_{12}=d_{23}=d_{34}=d_{45}=d_{56}=d_{61}=3, d_{13}=d_{46}=3\sqrt{3}, d_{25}=6$$



G
Figure 2

Under the gradient control law, the system has an unexpected equilibrium at $\hat{r}=[r_{12}^T,r_{23}^T,r_{34}^T,r_{45}^T,r_{56}^T,r_{61}^T,r_{13}^T,r_{46}^T,r_{25}^T]^T$

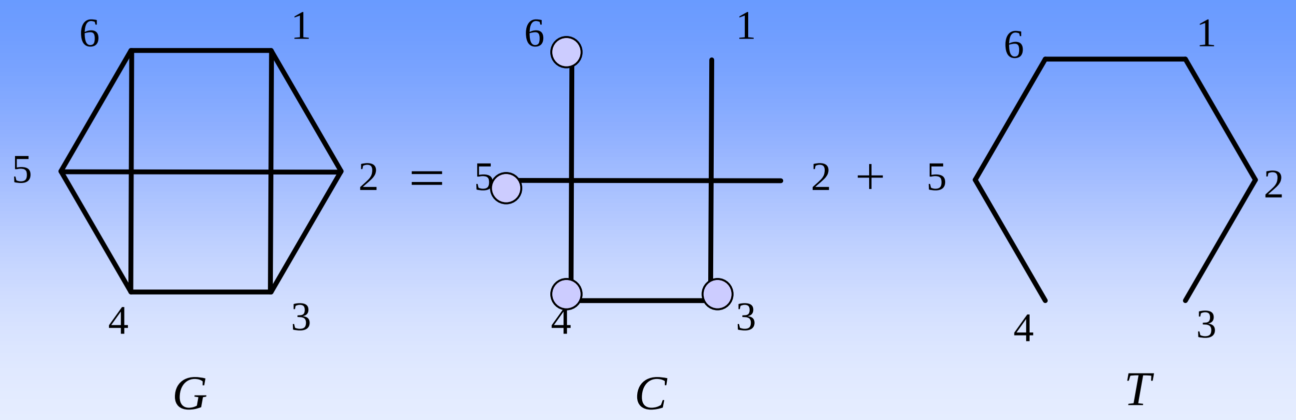
$$r_{12}^T=[-2.526,-1.429], r_{23}^T=[1.478,-3.403], r_{34}^T=[2.640,-2.173], r_{45}^T=[-2.896,0.197]$$

$$r_{56}^T=[-0.138,-3.707], r_{61}^T=[1.442,-3.101], r_{13}^T=[-1.409,-4.832], r_{25}^T=[1.222,-5.379], r_{46}^T=[=3.034,3.905]$$

we evaluate the eigenvalues of the linearized system about the undesired formation shape, all the eigenvalues are negative. So, the undesired formation shape is locally stable.

Stabilization by adaptive perturbation method

The desired formation is as the same as given before.



Stabilization by adaptive perturbation method

We choose $\omega_{31}=1,\ \omega_{43}=2.6,\ \omega_{52}=4.4,\ \omega_{64}=5.8$

Initial conditions are given as:

$$r_1(0)=[5,6]^T, r_2(0)=[8,8]^T, r_3(0)=[7,11]^T, r_4(0)=[4,12]^T, r_5(0)=[7,12]^T, r_6(0)=[7,9]^T.$$

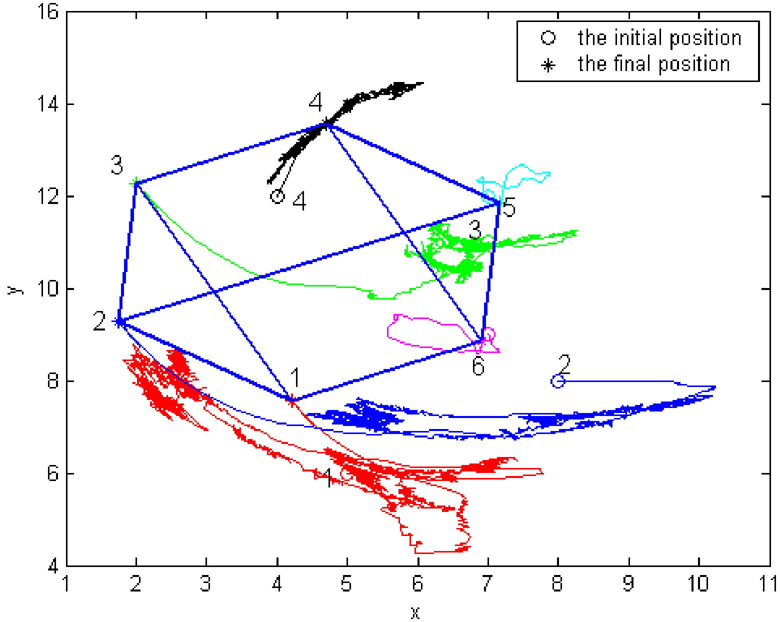


Figure 4: Movement trajectories of agents

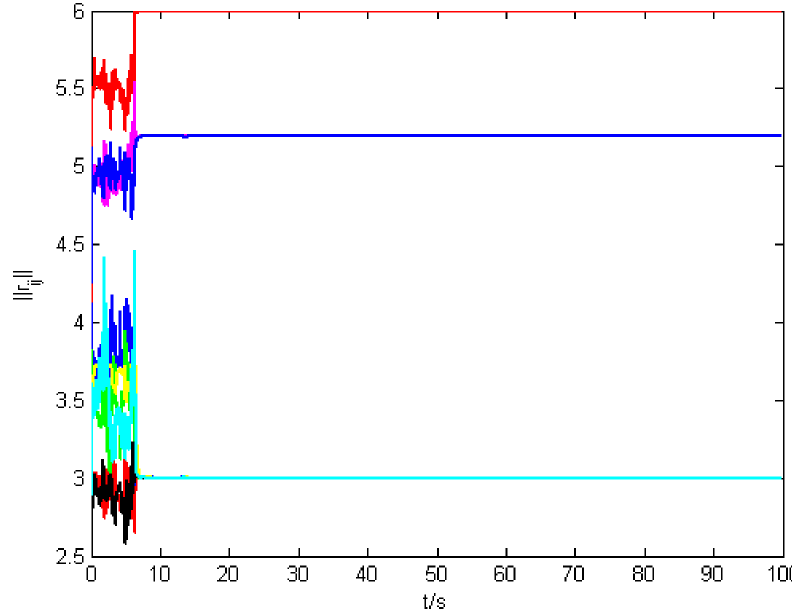


Figure 5: Distance between any two adjacent agents

Simulations

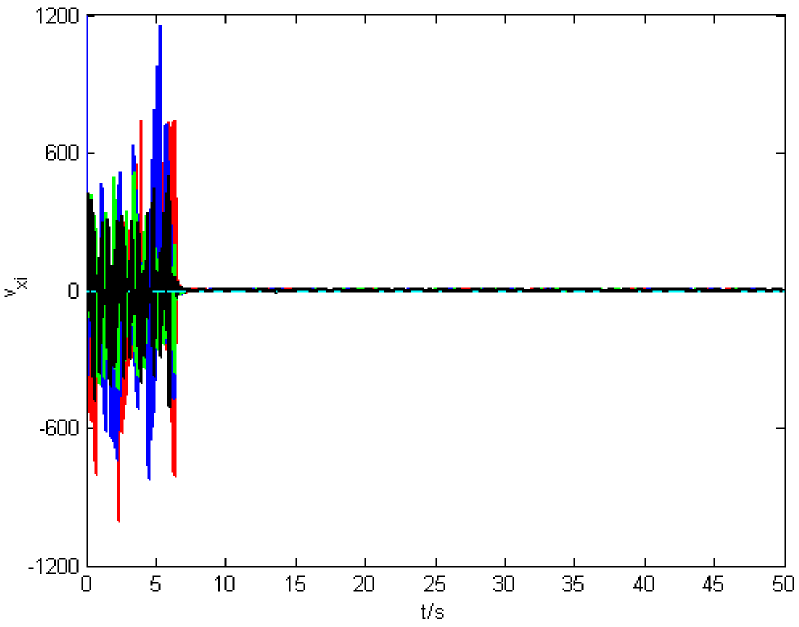


Figure 6: Agents' velocity along the *x*-axis

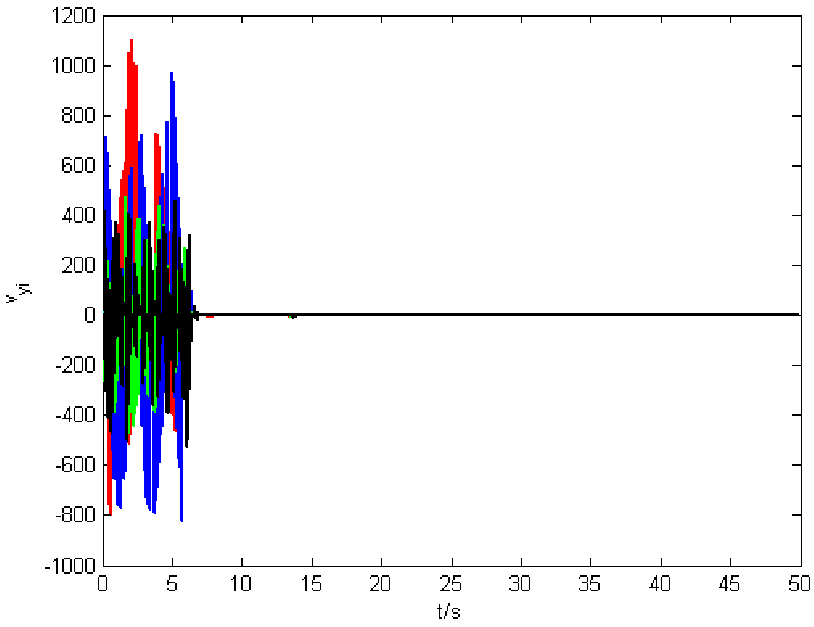


Figure 7: Agents' velocity along the *y*-axis

Simulations

The desired formation is the same.

$$w_{31}=1, w_{43}=2.6, w_{52}=4.4, w_{64}=5.8$$

Collinear initial condition:

$$r_1(0)=[1,1]^T, r_2(0)=[2,2]^T, r_3(0)=[3,3]^T, r_4(0)=[4,4]^T, r_5(0)=[5,5]^T, r_6(0)=[6,6]^T$$

From a collinear initial condition, the system remains collinear for ever under gradient control.

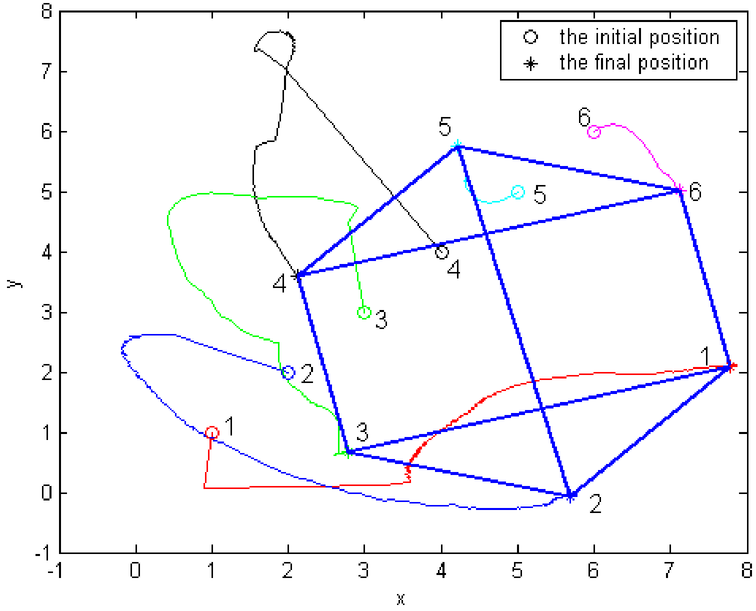


Figure 8: Movement trajectories of agents

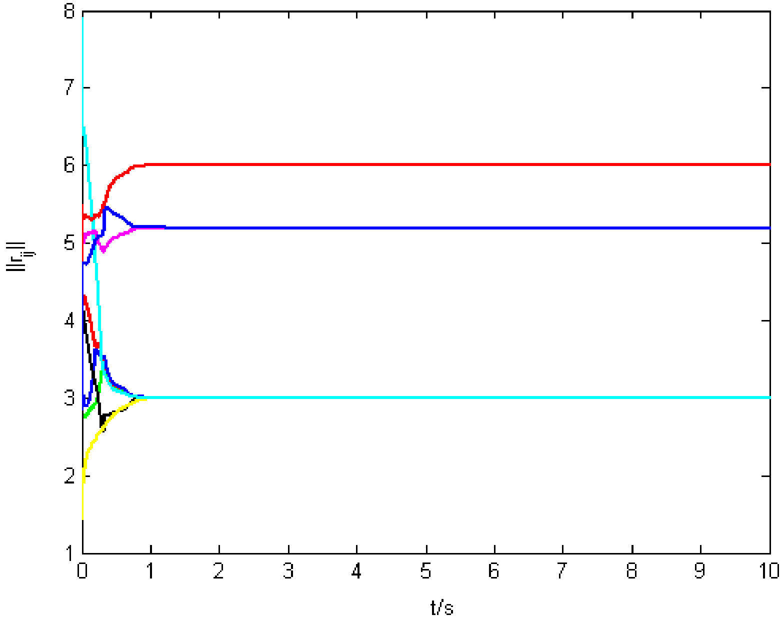


Figure 9: Distance between any two adjacent agents

Simulations

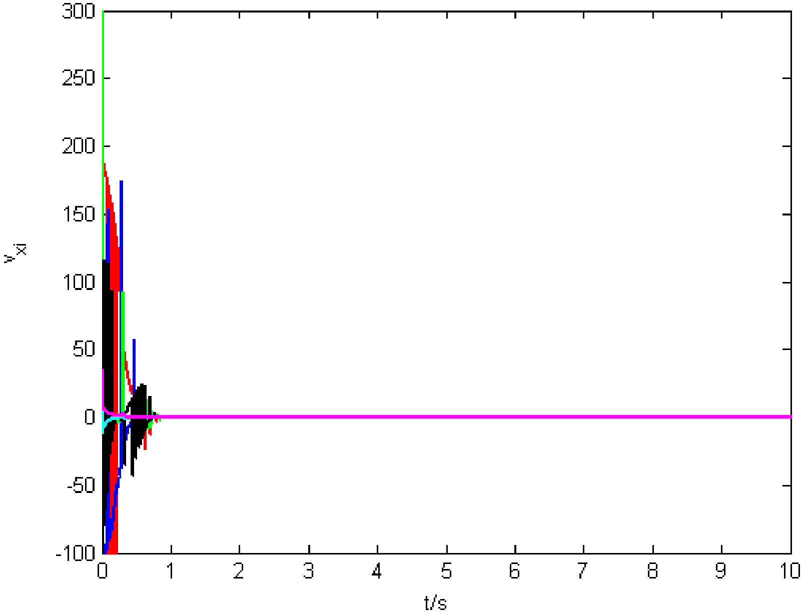


Figure 10: Agents' velocity along the *x*-axis

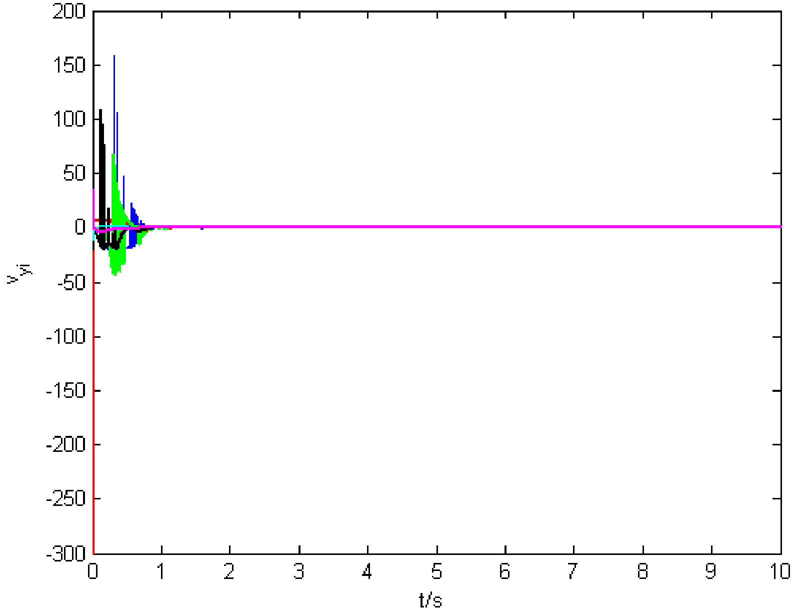


Figure 11: Agents' velocity along the *y*-axis

Simulations

A collinear formation cannot be stabilized by gradient control.

The desired distances are given by

$$d_{12}=d_{23}=d_{34}=d_{45}=d_{56}=3, d_{61}=15, d_{13}=d_{46}=6, d_{25}=9$$

We choose $w_{31}=1, w_{43}=2.6, w_{52}=4.4, w_{64}=5.8$

Initial condition:

$$r_1(0)=[1,11]^T, r_2(0)=[12,0]^T, r_3(0)=[10,13]^T, r_4(0)=[1,2]^T, r_5(0)=[10,2]^T, r_6(0)=[10,5]^T.$$

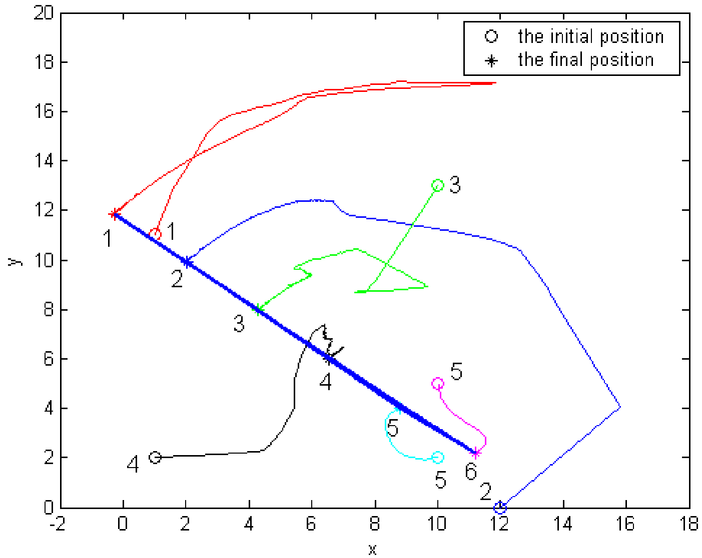


Figure 12: Movement trajectories of agents

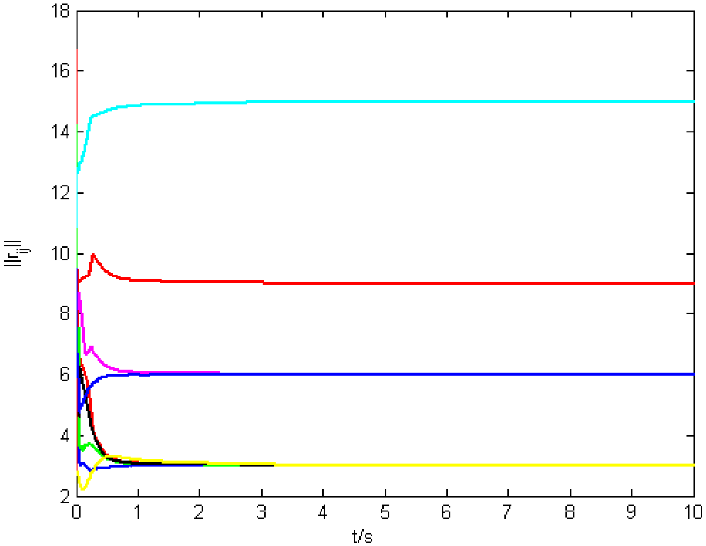


Figure 13: Distance between any two adjacent agents

Simulations

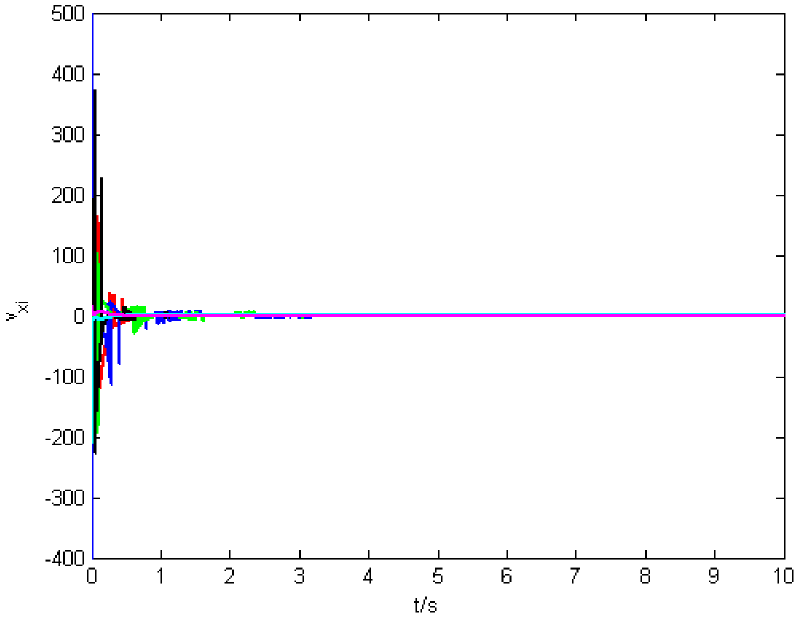


Figure 14: Agents' velocity along the *x*-axis

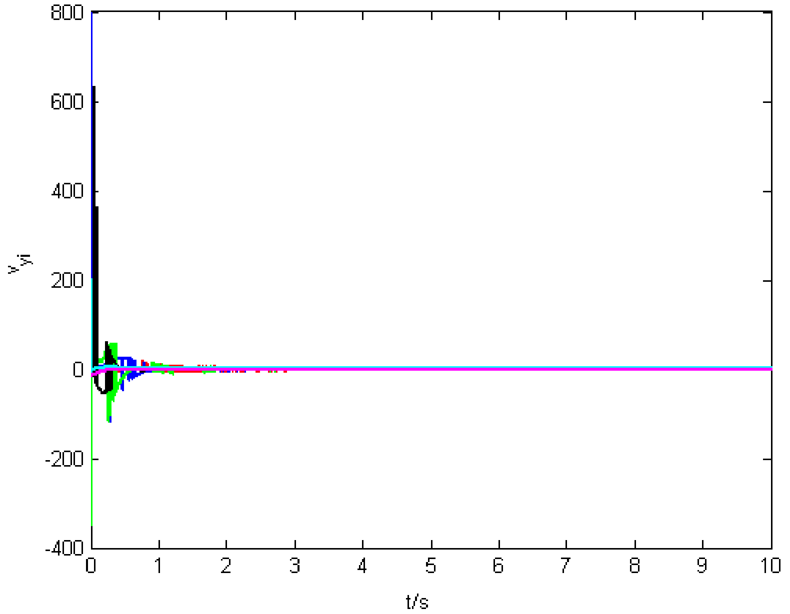


Figure 15: Agents' velocity along the *y*-axis

The end !

Thank you ！