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# Distributed Output Regulation of Multi-Agent Systems

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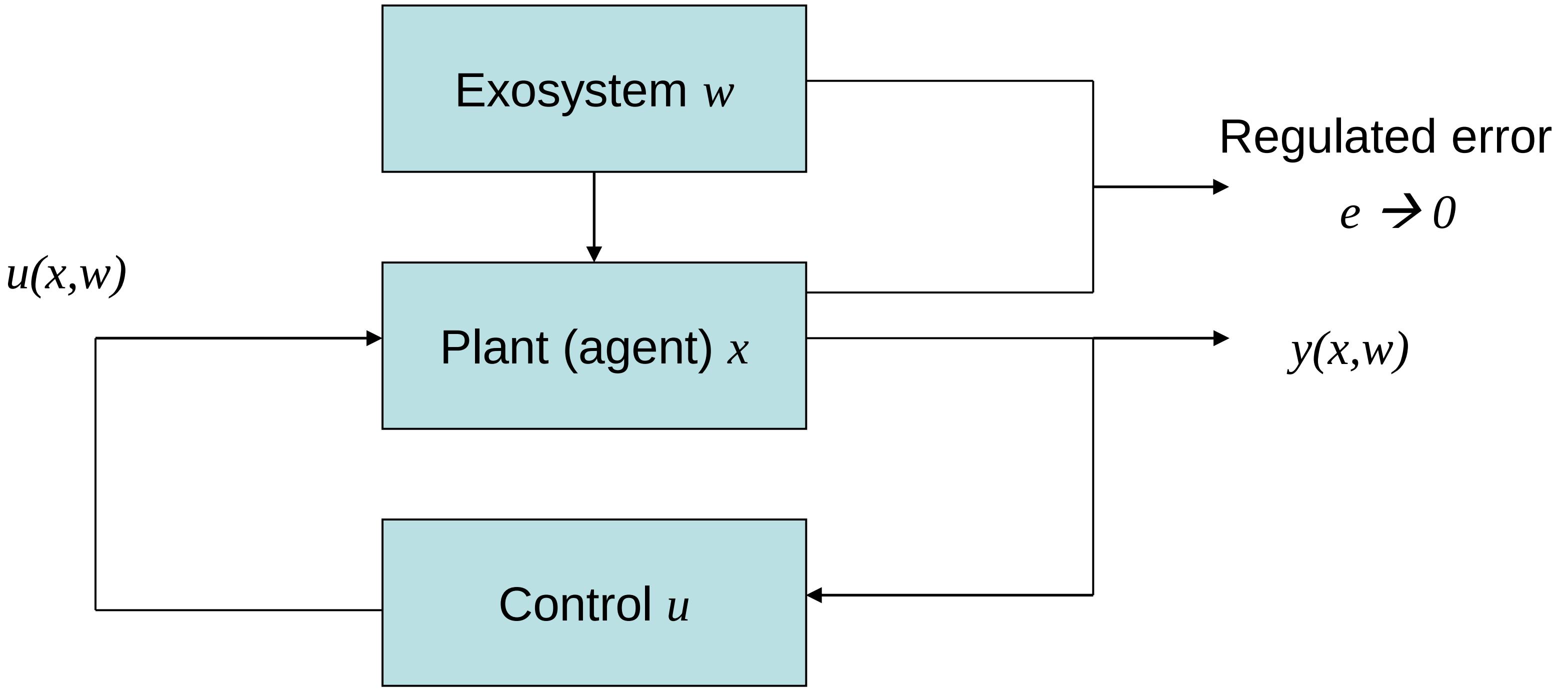
# Outline

1. Background
2. Problem Formulation
3. Analysis and Results
4. Conclusions

# 1. Background

- Multi-agent systems (MAS), a model for complex large-scale system & distributed information collection and control → communication, computation, control, ⋯
- Problems: consensus, formation, coverage, …
- Leader-follower or leaderless coordination
- Connectivity: fixed or switched interaction topology to describe information flow

# Output regulation




Applications: asymptotical tracking, disturbance rejection …

# Formulation

- Reference inputs or disturbances generated by an autonomous differential equation called exosystem.
- Output regulation (OR): design a control to make the regulated output $e \rightarrow 0$
- Stabilization: a special case of OR

# Problems

- Mathematical description: regulator equation (RE) + internal stability
- Robust OR → solvability of RE with uncertainty
- Internal model principle for robust OR: regulation controller should incorporate a suitably reduplicated model of the dynamic structure (internal model) of the exosystem (Wonham 1976)

# Our recent works (MAS)

- Distributed output regulation for (active) leader-following coordination (JSSC'09; Automatica'08; ICCA'10; IEEE TAC'10)
- Set coordination: static set aggregation (Automatica'09), containment (CDC'10, IEEE TAC'11), set optimization (ACC'11)
- Multi-agent coverage: static coverage (IFAC'08), sweep coverage (CCC'11)…

# 2. Problem Formulation

Distributed output regulation (DOR) model for leader-follower coordination (by distributed control of each agent):

1) Exosystem → active leader …

- Dynamics different from those of agents
- Unmeasurable states

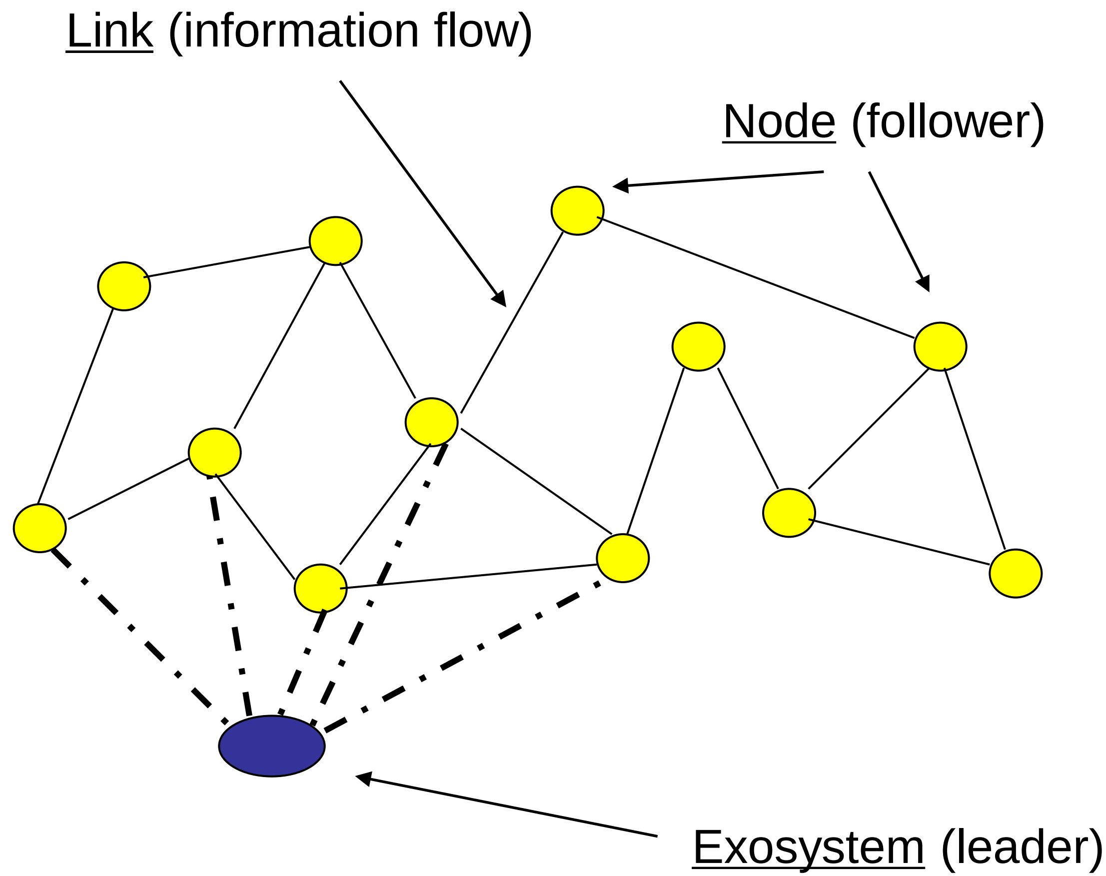
2) Plant → agent → a group of follower-agents with variable interaction topology

# Multiple Agents

**Distributed OR:**
Agents with distributed rules

↕

Exosystems: target, leader, disturbance …



# Multiple Agents

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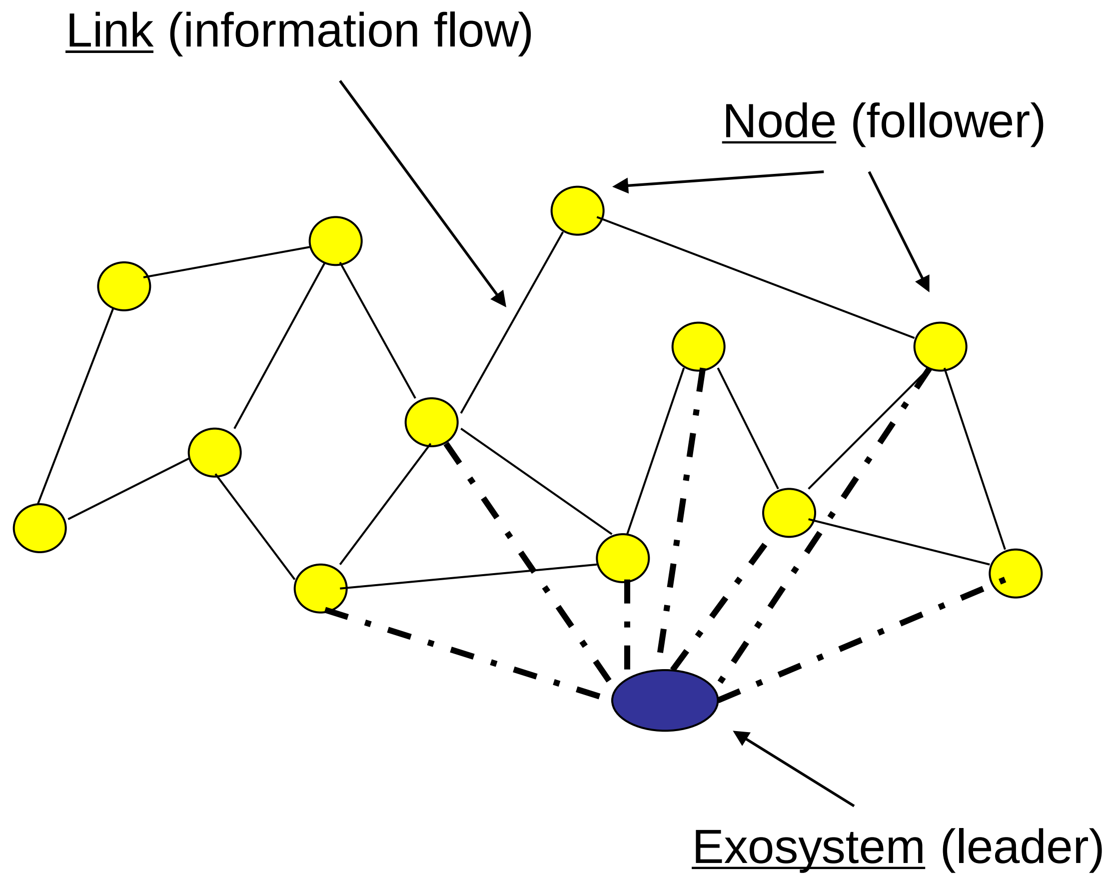


# Linear case

- Exosystem:

  $dw/dt = \Gamma w$; $w \in R^k$ , $y_0 = Fw \in R^l$ .

- Agent model:

  $dx_i/dt = Ax_i + Bu_i + Dw,\ x_i \in R^m\ ,\ y_i = Cx_i \in R^l,\ i = 1, \ldots, n$

- Regulated output: $e_i = y_i - y_0 \rightarrow 0$ as $t \rightarrow \infty$
- Network topology: (1) fixed and directed graph, or (2) switching and undirected graph

# Standard Assumptions

- Connectivity of directed graph: exosystem (node 0) is globally reachable
- *(A, B)* is stabilizable
- Real parts of eigenvalues of the exosystem are nonnegative ($\sigma(\Gamma) \geq 0$, to avoid trivial discussion)
- Rank condition: $\Lambda(\Gamma)$ spectrum of $\Gamma$

$$\text{rank} \begin{pmatrix} A - \lambda I & B \\ C & 0 \end{pmatrix} = n + q, \quad \lambda \in \Lambda(\Gamma)$$

# 3. Analysis and Results

- Stability + solvability of RE of MAS
- Switching topology → switching RE, common Lyapunov function
- Distributed design: neighbor-based rule, not only existence analysis, but also construction of distributed feedback
- Existing IM-based results with special topology: each agent can get the information of the exosystem (Gazi, et al, 2004, ⋯).

# Feedback Design

- Neighbor-based control:
  - **Static feedback (SF):**

$$u_i = K_z z_i + K_x x_i$$

  - **Dynamic feedback (DF):**

$$\begin{cases} u_i = K_z z_i + K_x x_i + K_v v_i \\ \dot{v}_i = E_z z_i + E_x x_i + E_v v_i \end{cases}$$

- Relative error ($N_i$: neighbor set of agent $i$):

$$z_i = \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + \mathbb{1}_{(i \in \mathcal{N}_0)} a_{i0}(x_i - C^+ F w)$$

# Regulation equations

- Closed loop system

$$\begin{cases} \dot{\xi} = A_c \xi + B_c w \\ \dot{w} = \Gamma w \\ e = (I_N \otimes C)x - (\mathbf{1} \otimes F)w \end{cases}$$

$x=(x_1, \ldots x_n)$ : state

$\xi=x$ in static feedback

$\xi=(x,v)$ in dynamic feedback

- Regulation constraints

$$\begin{cases} X_c \Gamma = A_c X_c + B_c \\ C_c X_c = F_c = \mathbf{1} \otimes F \end{cases}$$

# Static feedback

Limitation of state feedback: not robust, limited cases

**Result (sufficient conditions):** static feedback works if

1. $C$ is of rank $n$ ;
2. there is matrix $K$ such that $C^{-1}F\Gamma=(A+BK)C^{-1}F+D$ holds

# Dynamic feedback

More freedom in DF → more control power: less restrictive and more robust.

1. Special design for special systems: control may be simple (above simple cases)
2. "Universal" design based on IM with $E_x=0$, $E_z=CG_2$, $E_v=G_1$ in the compensator: systematical and robust

# IM-based approach

Existence of effective IM: can be guaranteed by the rank condition.

Equivalence: necessary and sufficient conditions for the solvability of OR (under the proposed IM controller)

1. Distributed OR is solved for the whole agent system
2. OR is solved for each “virtual” agent (after transformation)
3. Rank condition holds

# Fixed topology

Internal Model (IM) approach: output regulation → stability.

Two steps in analysis of IM-based distributed control:

- ❖ Step 1: transformation based on graph of the network topology → "virtual agent"
- ❖ Step 2: simultaneous control of "virtual agents"

# Switching topology

**Basic result**: there are common Lyapunov function (CLF) and common regulation matrix (CRM) $X_C$ → switching distributed OR is solved, where

$$\begin{cases} X_c\Gamma = A_c X_c + B_c \\ C_c X_c = F_c = \mathbf{1} \otimes F \end{cases}$$

Check the existence of CRM in some cases…

# Switching topology (2)

Two more steps for OR with switching topology

- ❖ Step 3: check and find CRM → CRM-based transformation
- ❖ Step 4: CLF based on the transformation → distributed design

# Main results

*Key: internal model construction → simultaneous control laws → CLF (based on CRM).*

- Fixed topology: IM-based (dynamic) feedback design
- Switching topology → still an open problem for general linear systems

# Example

5 followers: $dx_i/dt = Ax_i + Bu_i + Dw,\ y_i = Cx_i$

1 leader (exosystem): $dw/dt = \Gamma w,\ y_0 = Fw$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \Gamma = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \ D = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

# Topology

Interaction topology of MAS described by $H=L+A$

*$L$: Laplacian; $A$: adjacency matrix of the leader*

A known result: $H$ is of full rank if the graph (containing the leader) is connected

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

# Control design

Internal model:

$$G_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

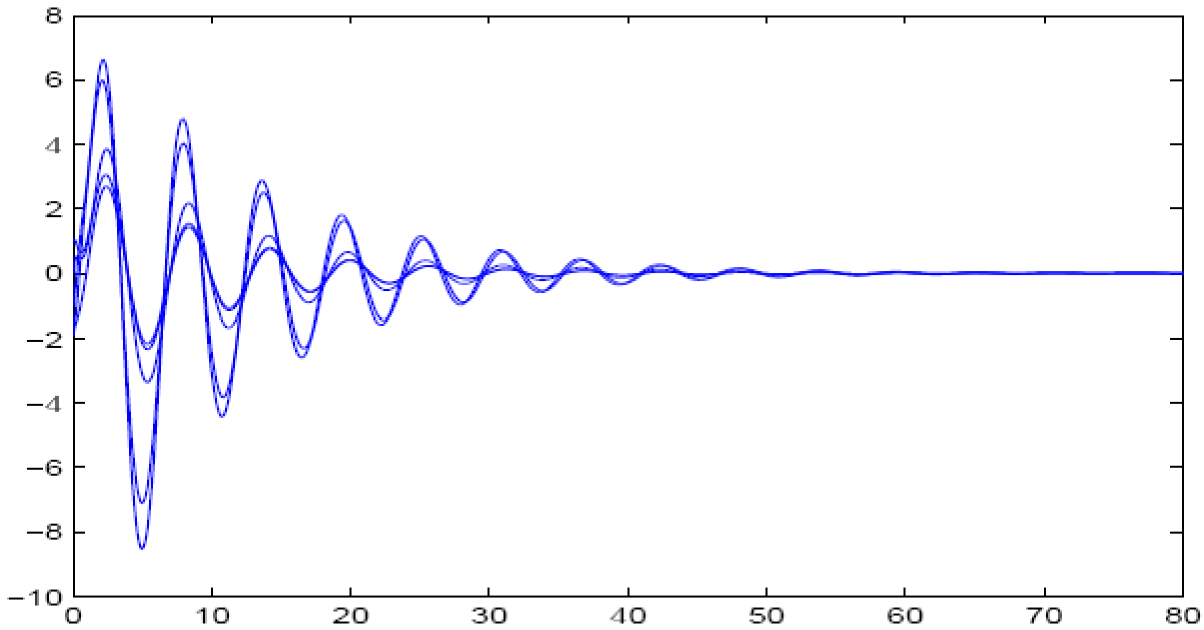
IM-based control:

(using Matlab)

$$\begin{cases} u_i = (k_1 \quad k_2) z_i + (k_{\upsilon 1} \quad k_{\upsilon 2} \quad k_{\upsilon 3}) v_i \\ \dot{v}_i = G_1 v_i + G_2 C z_i, \quad v_i \in \mathbb{R}^3 \end{cases}$$

$$(k_1 \; k_2 \; k_{\upsilon 1} \; k_{\upsilon 2} \; k_{\upsilon 3}) = -(4.2948 \;\; 3.0967 \;\; 1 \;\; -0.6316 \;\; 2.6128)$$

# Simulations




Regulated errors of 5 follower agents

# 4. Conclusions

- Distributed output regulation: leader-follower multi-agent systems → coordination between heterogeneous agents …
- Linear systems → nonlinear or stochastic systems (with assumptions to guarantee the solvability of RE) → more complex models with communication constraints and hybrid structures …

# Thank You!