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Identification and Adaptive Tracking Control with Set-Valued Observations

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Outline

- Background
- Identification
- Adaptive tracking control
- Concluding remarks

1. Background

Identification and Filtering

- **Model:**
  - $\dot{x} = Ax + Bu$
  - $y_t = \sum_{i=1}^{n} a_i y_{t-i} + \sum_{i=1}^{m} b_i u_{t-i} + d_t$
- **Algorithm: LS、SG、MLE、KF、…**
- **Key precondition :** ***The output is known !***

(or with noise)

1. Background

Feedback Control

- **System:**

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

Controllers:

$$\mathbf{u} = \mathbf{K}\mathbf{x}; \quad \mathbf{u} = \mathbf{K}\mathbf{y}$$

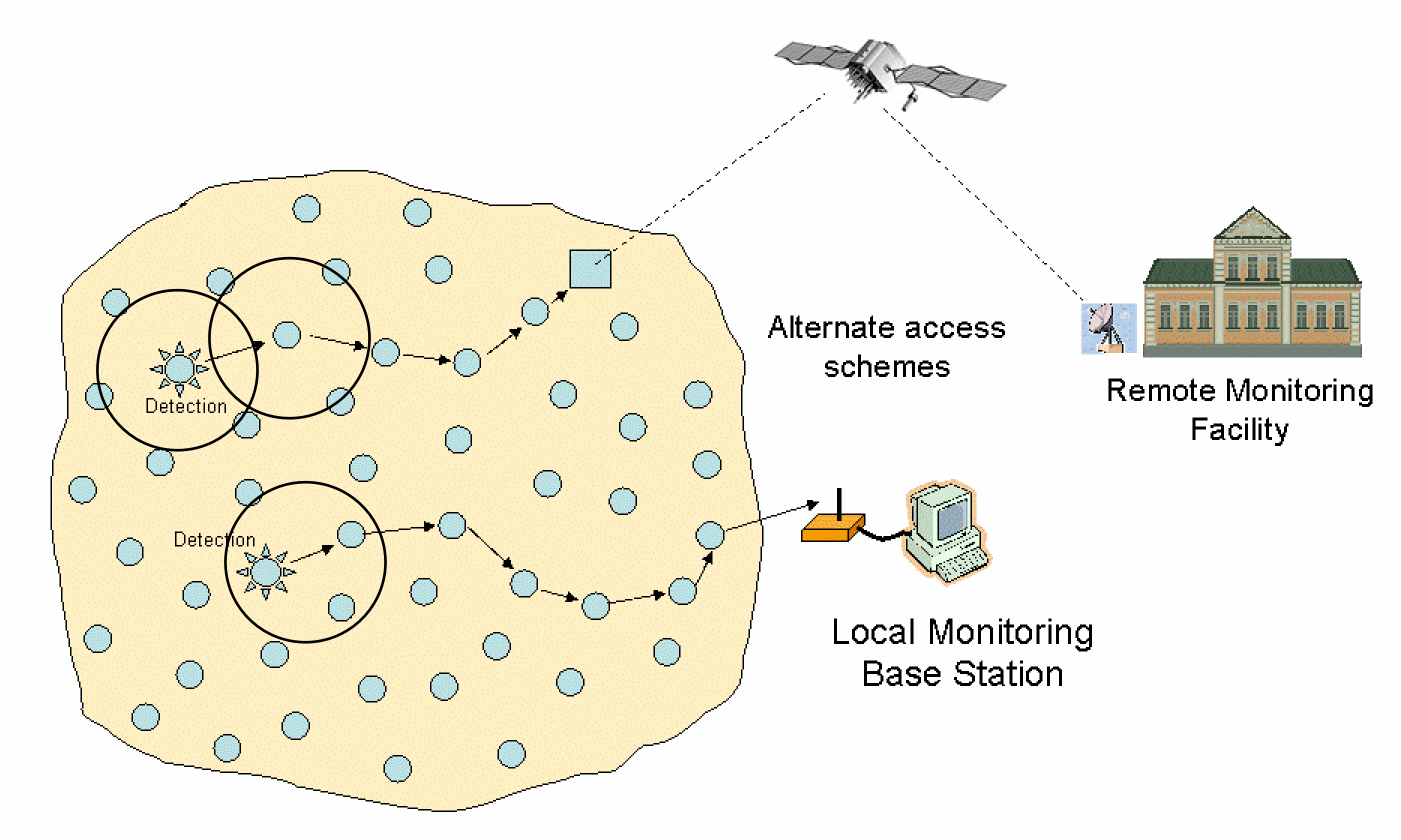
$$\begin{cases} \dot{\hat{\mathbf{x}}} = \mathbf{B}\mathbf{y} + \mathbf{F}\mathbf{u} \\ \mathbf{u} = \mathbf{K}\hat{\mathbf{x}} + \mathbf{H}\mathbf{y} \end{cases}$$

- **Key precondition:**

The state or output is known !

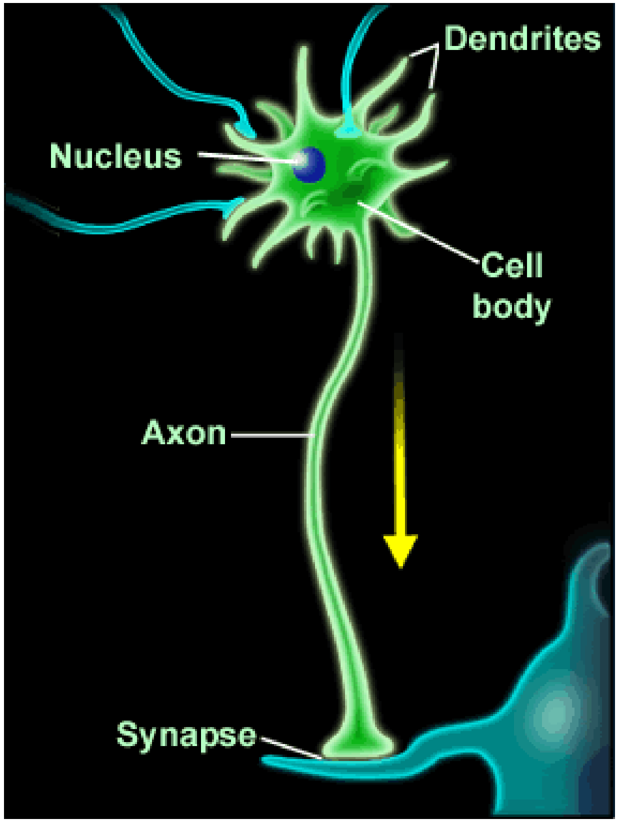
1. Background

I. Wireless sensor network



1. Background

II. Neural network model

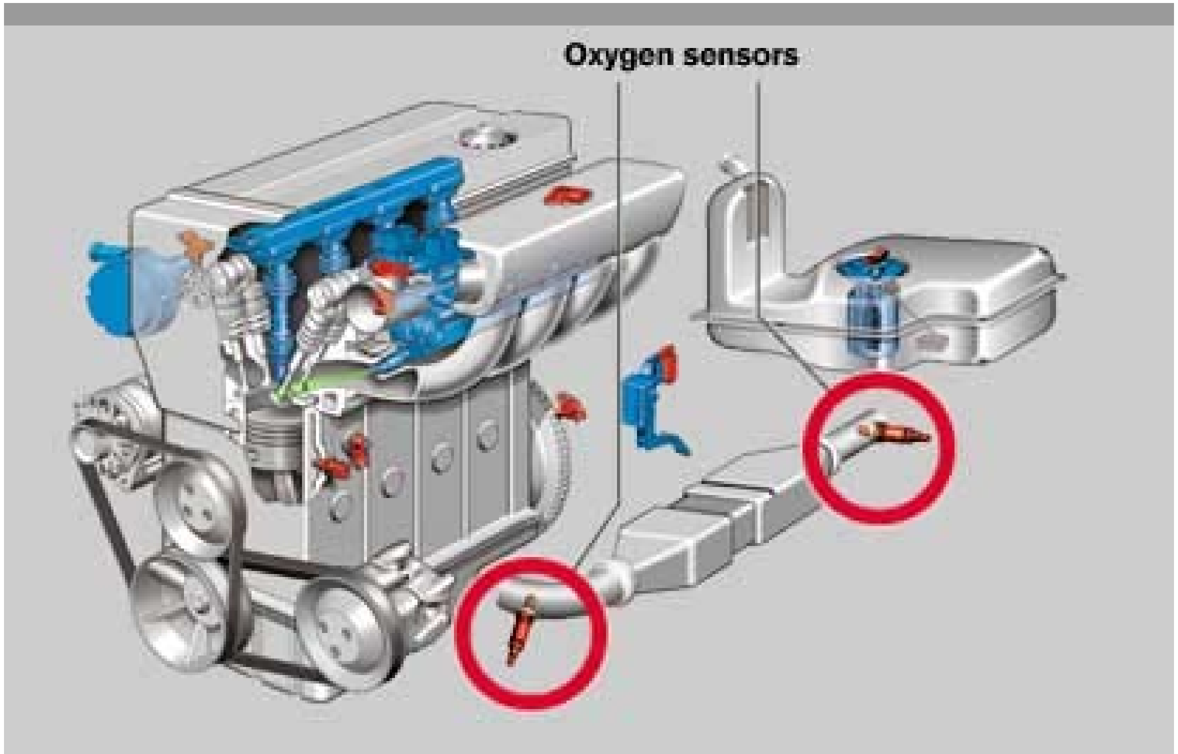


- Excitation and inhibition
- Threshold exists

$$s = S\left(\sum_{j=1}^{n_1} w_j x_j + \sum_{k=1}^{n_2} \tilde{w}_k \tilde{x}_k - C \right),$$

1. Background

III. Oxygen sensors



Oxygen Sensors in Automotive Exhaust Gas Aftertreatment Systems (BOSCH)

1. Background

- Practical systems

  Small

  Cost effective

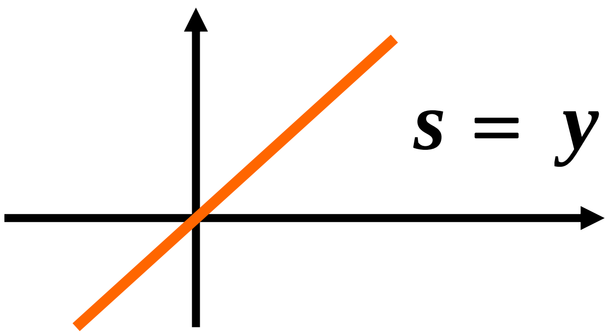
- Challenge

  Limit information

1. Background

Set-valued sensors:

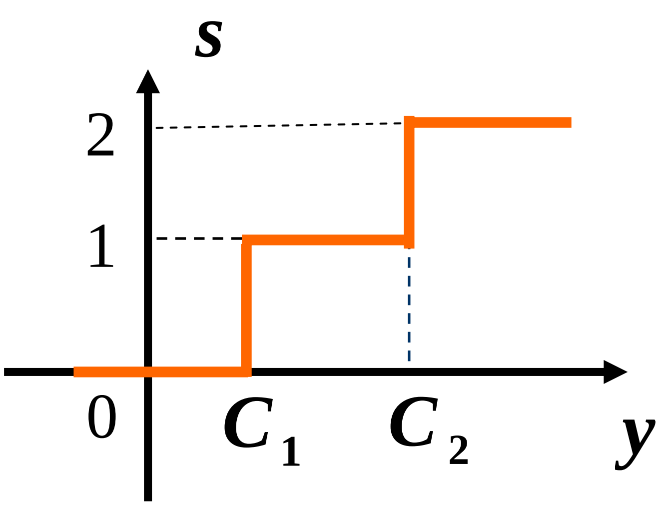
- Accurate sensor



- Set-valued sensor

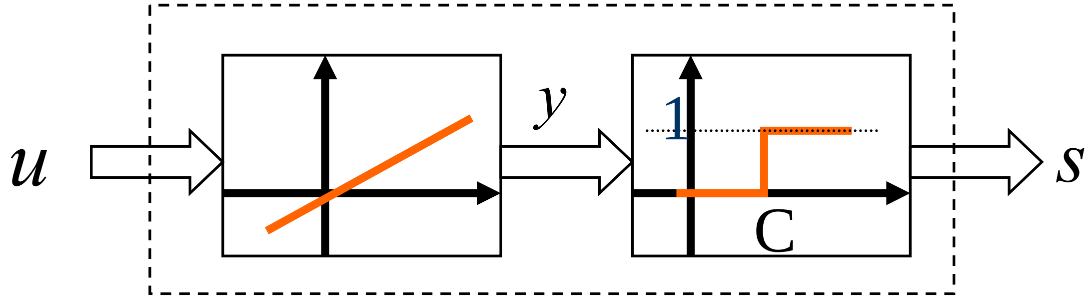
$$s=\begin{cases}2, & \text{if } y>C_2;\\ 1, & \text{if } C_1<y\le C_2;\\ 0, & \text{if } y\le C_1.\end{cases}$$

Threshold



1. Background

System model:



$$y(t) = P(y, u, \theta) + d(t),$$

$$s(t) = S(y(t))$$

Estimate θ with the information of u and s

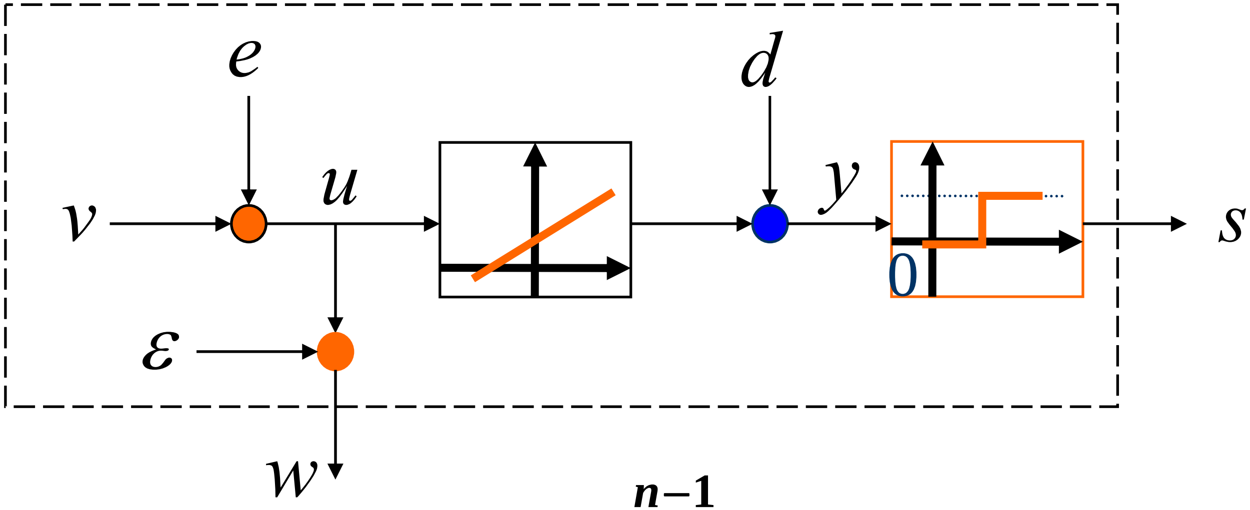
2. Identification

- **Linear systems with set-valued observations**

Algorithm

Properties

2.1. Linear systems



Model:

$$y(k) = \phi^T(k)\theta = \sum_{i=0}^{n-1} a_i u(k-i) + d(k),$$

$$s(k) = \begin{cases} 0, & y(k) > C; \\ 1, & y(k) \le C. \end{cases}$$

$$\theta = [a_0, \ldots, a_{n-1}]^T, \ \phi(k) = [u(k), \cdots, u(k-n+1)]^T.$$

2.1. Linear systems

Denote

$$Y(j) = [y((j-1)n+1), \cdots, y(jn)]^T$$
$$\Phi(j) = [\phi((j-1)n+1), \cdots, \phi(jn)]^T$$
$$D(j) = [d((j-1)n+1), \cdots, d(jn)]^T$$
$$S(j) = [s((j-1)n+1), \cdots, s(jn)]^T$$

Let

$$\xi(N) = \frac{1}{N}\sum_{j=1}^{N} S(j),$$

2.1. Linear systems

$$\xi(N) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{I}_{\{D_j \leq \mathbf{C}_n - \Phi\theta\}} \rightarrow \xi = \mathbf{F}(\mathbf{C}_n - \Phi\theta)$$

2.1. Linear systems

$$\xi(N) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{I}_{\{D_j \le \mathbf{C}_n - \Phi\theta\}} \longrightarrow \xi = \mathbf{F}(\mathbf{C}_n - \Phi\theta)$$

$$\begin{pmatrix} v_n & v_{n-1} & \cdots & v_1 \\ v_1 & v_n & & v_2 \\ & \ddots & \ddots & \\ v_{n-1} & v_{n-2} & \cdots & v_n \end{pmatrix}$$

C, F is known

2.1. Linear systems

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C, F is known

$$\Phi^{-1}[C_n - \mathrm{F}^{-1}(\xi(N))] \rightarrow \theta$$

2.1. Linear systems

Full rank condition:

Φ is full rank if and only if

$$\gamma_k = \sum_{j=1}^{n} v_j e^{-i\omega_k j} \neq 0,$$

where $\omega_k = {2\pi k}/{n}, k = 1, \cdots, n.$

Frequency is not 0

2.1. Linear systems

Full rank signal:

u is periodic with one period $v = (v_1, \cdots, v_n)$ full rank.

$$\begin{pmatrix} v_n & v_{n-1} & \cdots & v_1 \\ v_1 & v_n & & v_2 \\ & \ddots & \ddots & \\ v_{n-1} & v_{n-2} & \cdots & v_n \end{pmatrix}$$

2.1. Linear systems

Assumption A1:

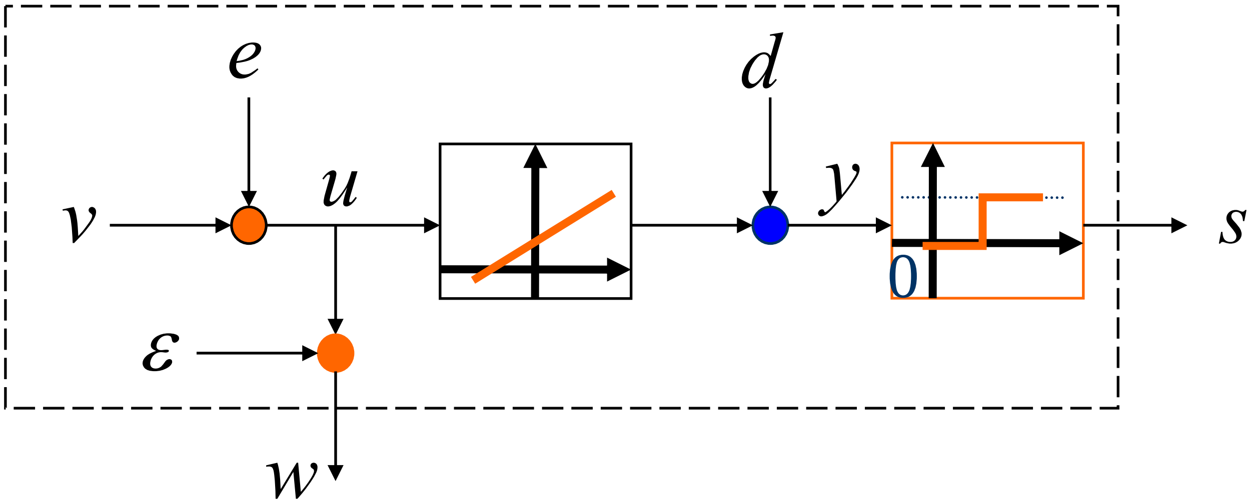
$\{d(k)\}$ is i.i.d with $\mathrm{E}d(1)=0,\ \sigma_{\mathrm{d}}^{2}=\mathrm{E}|d(1)|^{2}<\infty$,
its d.f. $F(\cdot)$ is known and continuous ly derivative
with inverse $F^{-1}(\cdot)$ and bounded density function $f(\cdot)$.

Basic algorithm:

$$\hat{\theta}(N)=\Phi^{-1}[\mathrm{C}-\mathrm{F}^{-1}(\xi(N))]\rightarrow\theta,\ \text{w.p.1.}$$

（Wang, Zhang & Yin, IEEE TAC， 2003)

2.1. Linear systems



- ✓ Unknown threshold
- ✓ Unknown d.f. of *d*
- ✓ Different noise: input noise

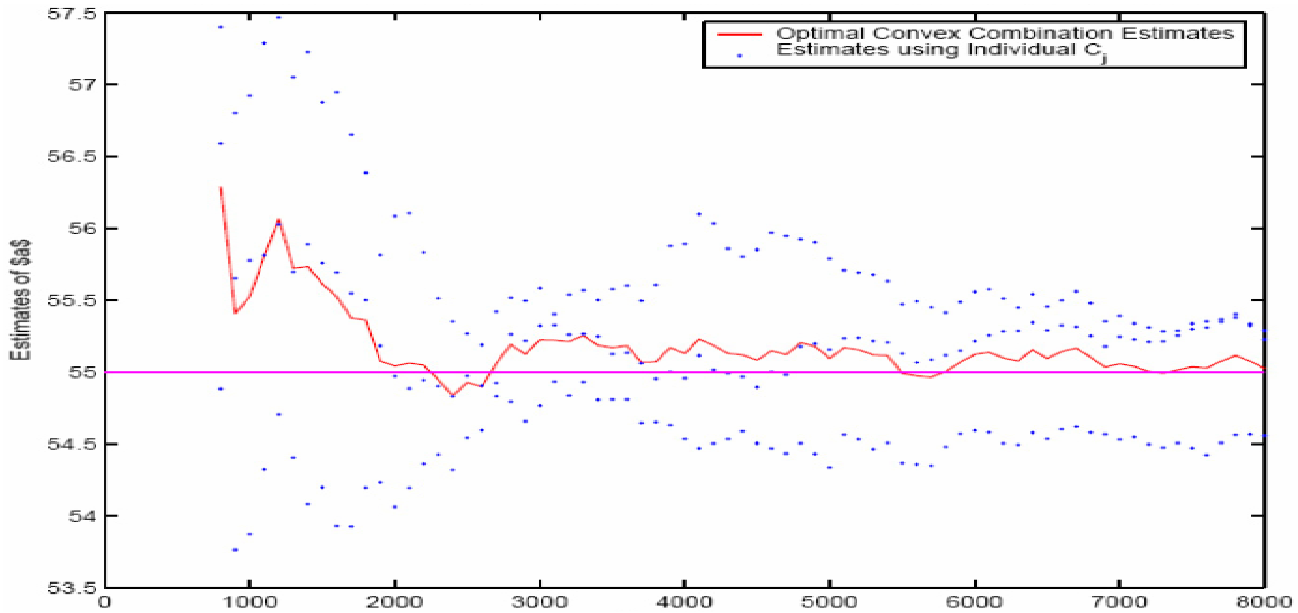
（**Wang, Yin, Zhao & Zhang, IEEE TAC， 2008)**

2.1. Linear systems

Set-valued observations:

$$\hat{\theta}(N) = \sum_{k=1}^{l} c_k(N)\hat{\theta}_k(N)$$

$$\min \sigma^2(N), \text{subject to} \sum_{k=1}^{l} c_k(N) = 1$$



2.1. Linear systems

Properties:

(Zhao, Zhang, Wang & Yin, SIAM J. on Control and Optimization, 2010)

Convergent

$$\theta(N) \to \theta \qquad w.p.1$$

Convergence speed

$$\sigma^2(N) = O(1/N).$$

Asymptotically efficient

$$N[\sigma^2(N) - \sigma^2_{CR}(N)] \to 0.$$

2.1. Linear systems

Time and space complexity (Information):

Requirement $\sigma^2(m,N) = O(1/N) < \varepsilon.$

$\eta(m)/N$

Time $N = N(m,\varepsilon).$

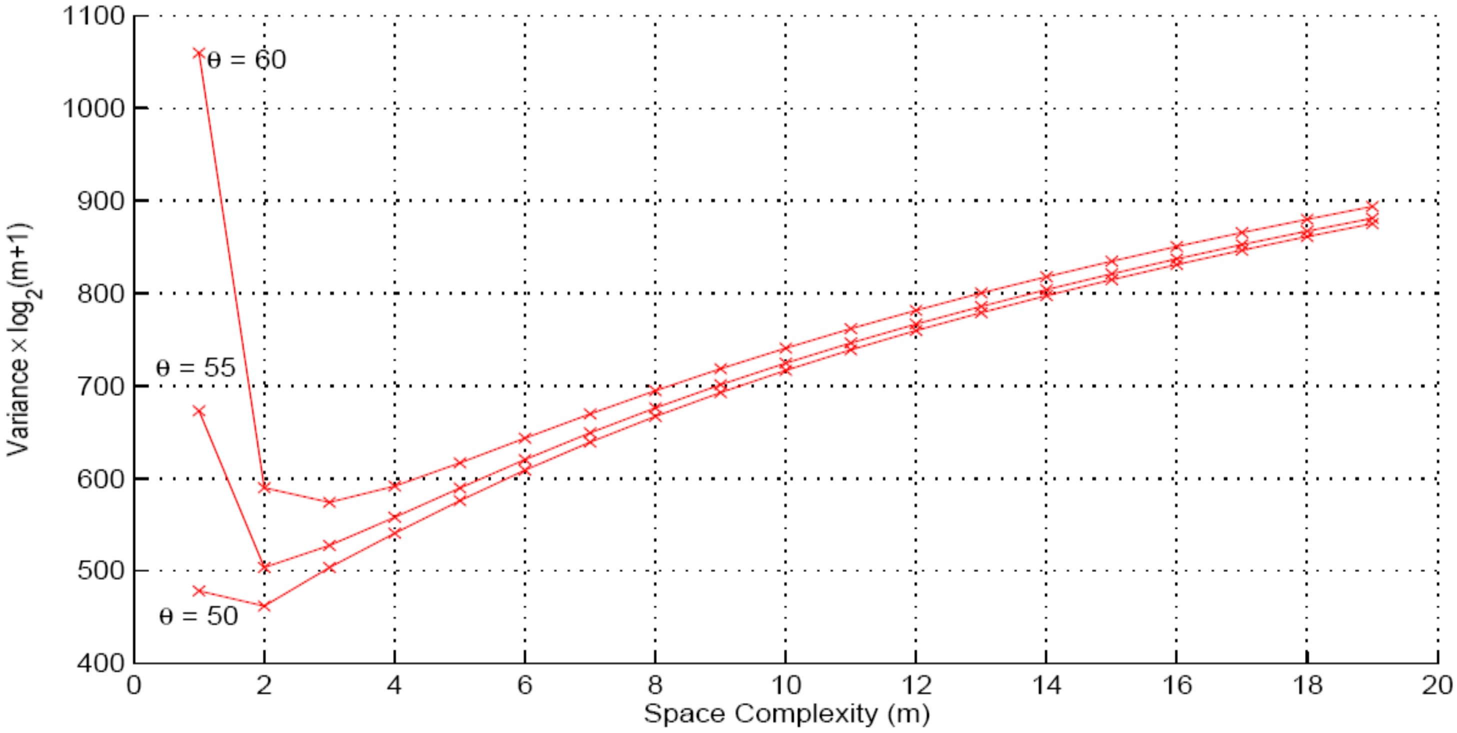
Information $R = N\log(m+1) = R(m).$

Minimization problem

$$\min_m R(m), \text{ with } \sigma^2 < \varepsilon$$

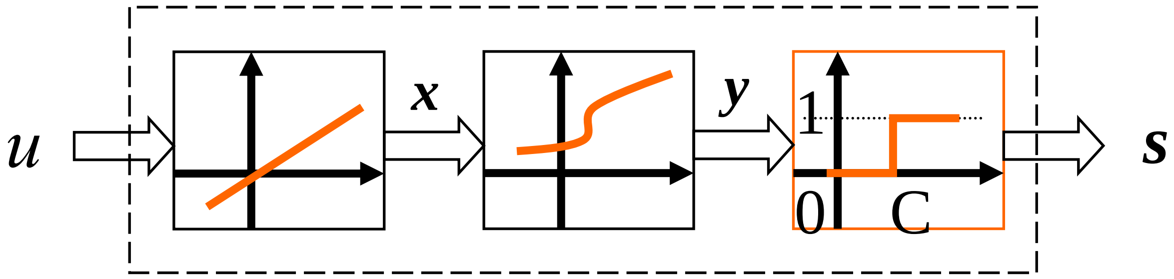
2.1. Linear systems

Time and space complexity:



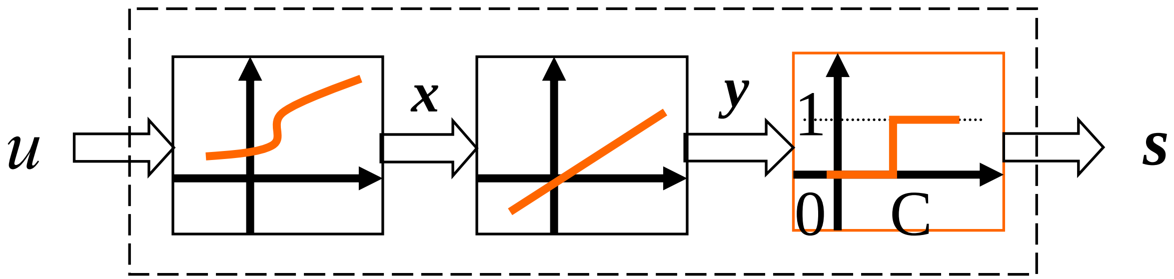
2.2. Related works

Wiener system: (Automatica, 2007)



$L(\theta) \rightarrow N_1(\eta) \rightarrow N_2$

Hammerstein system: (SICON, 2010)



$N_1(\eta) \longrightarrow L(\theta) \longrightarrow N_2$

Unbiased and efficient properties

Adaptive Control ???

3. Adaptive tracking control

Model:

$$y(k) = \phi^T(k)\theta = \sum_{i=0}^{n-1} a_i u(k-i) + d(k),$$

$$s(k) = \begin{cases} 0, & y(k) > C; \\ 1, & y(k) \le C. \end{cases}$$

Estimate:

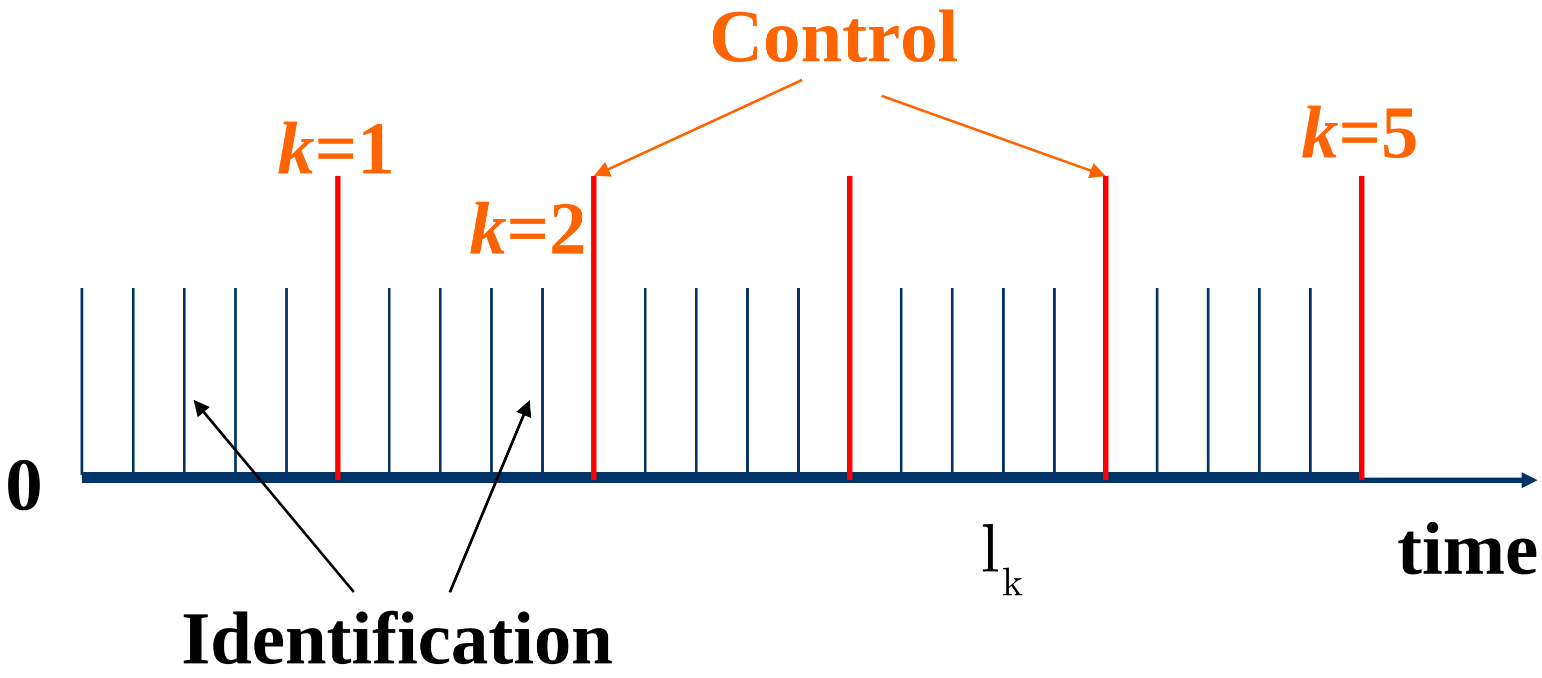
$$\theta = [a_0, \ldots, a_{n-1}]^T,$$

Track periodic target:

$$y = [y_1, \ldots, y_n]$$

$$\lim_{k\to\infty} \mathrm{E}(\mathrm{y}(\mathrm{k}) - \mathrm{y})^2 = \mathrm{Ed}_1^2$$

3. Adaptive tracking control



3. Adaptive tracking control

Model:

$$y(k) = \phi^T(k)\theta = \sum_{i=0}^{n-1} a_i u(k-i) + d(k),$$

$$s(k) = \begin{cases} 0, & y(k) > C; \\ 1, & y(k) \le C. \end{cases}$$

Estimate:

$$\theta = [a_1, \ldots, a_n]^T,$$

Track periodic target:

$$y = [y_1, \ldots, y_n]$$

3. Adaptive tracking control

Target:

$$Y = \begin{pmatrix} y_m & y_{m-1} & \cdots & y_1 \\ y_1 & y_m & & y_2 \\ & \ddots & \ddots & \\ y_{m-1} & y_{m-2} & \cdots & y_m \end{pmatrix}$$

Estimate:

$$\mathbf{\Theta} = \begin{pmatrix} a_n & a_{n-1} & \cdots & a_1 \\ a_1 & a_n & & a_2 \\ & \ddots & \ddots & \\ a_{n-1} & a_{n-2} & \cdots & a_n \end{pmatrix}$$

Design:

$$\Phi = \begin{pmatrix} u_m & u_{m-1} & \cdots & u_1 \\ u_1 & u_m & & u_2 \\ & \ddots & \ddots & \\ u_{m-1} & u_{m-2} & \cdots & u_m \end{pmatrix}$$

3. Adaptive tracking control

m=n: $$\Phi\Theta = Y$$

Step 1: Control (time *k*)

$$\Theta(k-1),$$

$$\Phi(k) = Y\Theta^{-1}(k-1)$$

Step 2: Estimate (l_k)

$$\theta(k) = \Phi_{(k)}{}^{-1}F^{-1}(C - \xi_{(l_k)})$$

3. Adaptive tracking control

m=n:

$$\Phi\Theta = \mathrm{Y}$$

m<n:

$$\boldsymbol{\Theta} = \begin{pmatrix} b_m & b_{m-1} & \cdots & a_1 \\ b_1 & b_m & & a_2 \\ & \ddots & \ddots & \\ b_{m-1} & b_{m-2} & \cdots & b_m \end{pmatrix}$$

$$b_i = a_i + a_{i+m} + \ldots + a_{i+[n/m]}$$

3. Adaptive tracking control

m=n:

$$\Phi\Theta = Y$$

m>n:

$$\Theta = \begin{pmatrix} a_m & a_{m-1} & \cdots & a_1 \\ a_1 & a_m & & a_2 \\ & \ddots & \ddots & \\ a_{m-1} & a_{m-2} & \cdots & a_m \end{pmatrix}$$

$$\theta = [a_1, \ldots, a_n, 0, \ldots, 0]_m^T$$

3. Adaptive tracking control

$$\Phi\Theta = Y$$

$$\theta - \text{Minimum Phrase Condition}$$

⇓

$$\Theta - \text{Full rank}$$

$$\gamma = \sum_{i=1}^{n} a_i x^i \neq 0,$$

$$\because \ |x| \leq 1$$

$$\therefore \ x = e^{-\omega_k j}, \omega_k = 2\pi k / n, \ k = 1, \cdots, n.$$

3. Adaptive tracking control

m=n: $$\Phi\Theta = Y$$

Step 1: Control (time *k*)

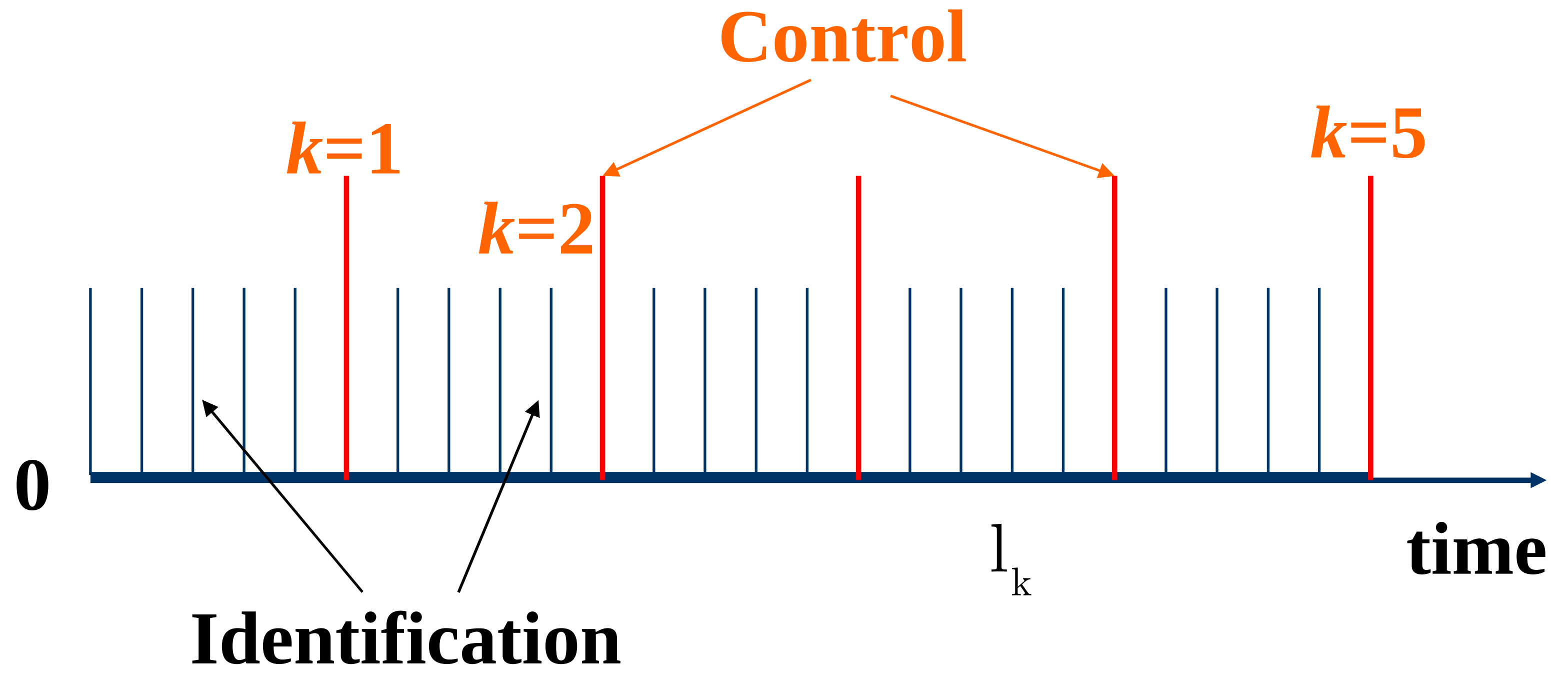
$$\Theta(k-1),$$
$$\Phi(k) = Y\Theta^{-1}(k-1)$$

Step 2: Estimate (l_k)

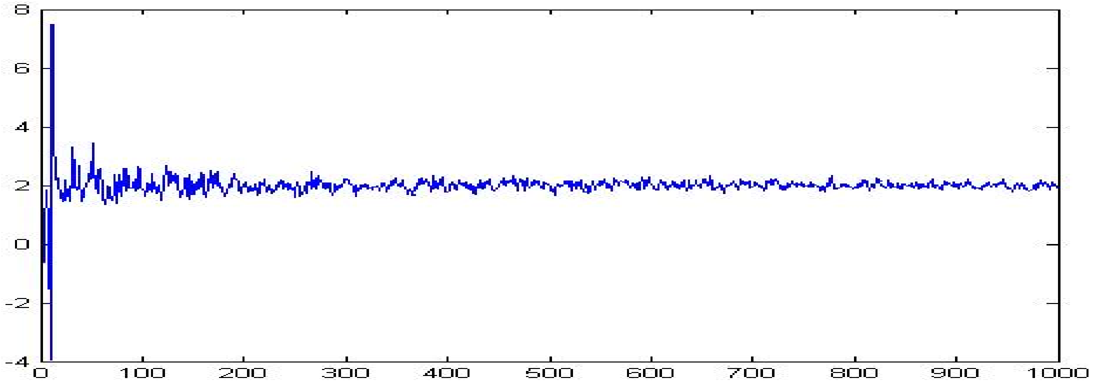
$$\theta(k) = \Phi_{(k)}{}^{-1}F^{-1}(C-\xi_{(l_k)})$$

$l_k = k$ (goes to infinity)

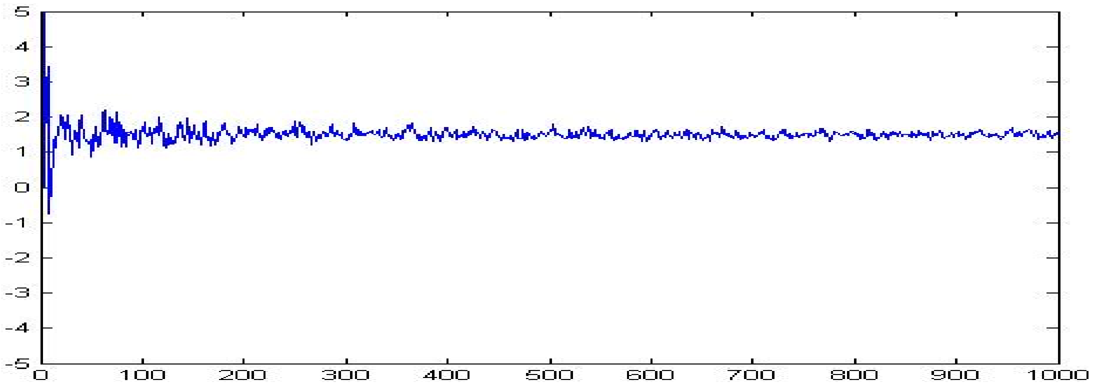
3. Adaptive tracking control



3. Adaptive tracking control



Tracking



Estimation

$$\theta = 1.5, \quad C = 2, \quad l_k = k$$

3. Adaptive tracking control

Result:

- Estimation (CR Lower Bound)
- Asymptotically optimal

$$\lim_{k\to\infty} E(y(k) - y)^2 = Ed^2(1)$$

3. Adaptive tracking control

Result:

- Estimation (CR Lower Bound)
- Asymptotically optimal

$$\lim_{k\to\infty} \mathbf{E}(\mathbf{y}(\mathbf{k})-\mathbf{y})^2 = \mathbf{E}\mathbf{d}^2(\mathbf{1})$$

$$\boldsymbol{\Theta} = \begin{pmatrix} b_m & b_{m-1} & \cdots & b_1 \\ b_1 & b_m & & b_2 \\ & \ddots & \ddots & \\ b_{m-1} & b_{m-2} & \cdots & b_m \end{pmatrix}$$

$$b_i = a_i + a_{i+m} + \ldots + a_{i+[n/m]}$$

4. Concluding remarks

Systems:

- Set-valued (Fixed C or not)

Identification and Control:

- Possible

Thanks！