Adaptive Control and Applications

Shuzhi Sam Ge

Robotics Institute (RI), and
School of Computer Science and Engineering,
University of Electronic Science and Technology of China (UESTC)
http://www.isit.org.cn

Social Robotics Lab, Interactive Digital Media Institute, &
Department of Electrical and Computer Engineering
National University of Singapore
http://robotics.nus.edu.sg/sge

Control of Uncertain Constrained Nonlinear Systems with Barrier Lyapunov Function

in Collaboration with

K.P. Tee, I2R, A*Star, Singapore

The Chernobyl Incident, 1986

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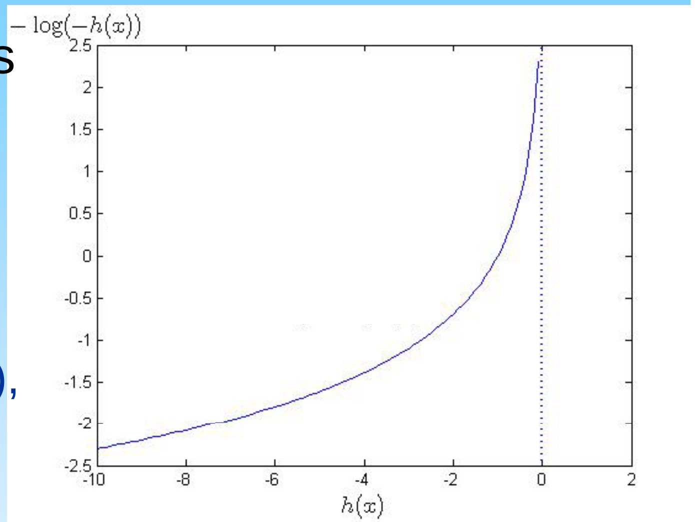
Objectives

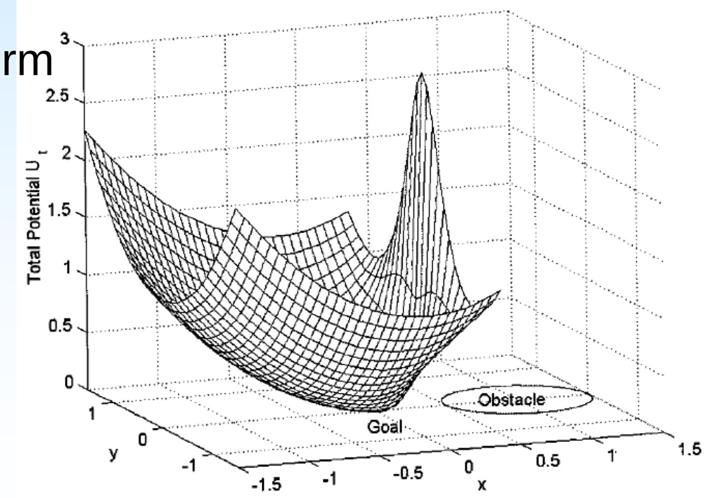
- To develop constructive and systematic methods of designing adaptive controllers for uncertain constrained nonlinear systems
- To provide technically rigorous framework for the use of Barrier Lyapunov Functions in ensuring constraint satisfaction
- To illustrate practical relevance through an application study

Barrier Lyapunov Functions

Barrier Functions

- A barrier function is a continuous function *b*(*x*) whose value increases to infinity as *x* approaches the boundary of the feasible region
- Constrained optimization problems
  - (Nocedal & Wright, 1999)
- Collision avoidance problem in robotics
  - (Rimon and Koditschek, 1992), (Ge and Cui, 2000), (Gazi and Passino, 2004), (Do, 2007)
- Barrier functions for control
  - (Ngo, Mahony & Jiang, 2005)
  - Considered known system in Brunowski normal form subjected to state constraint
  - States assumed to be constrained before being proved so





Barrier Lyapunov Function (BLF)

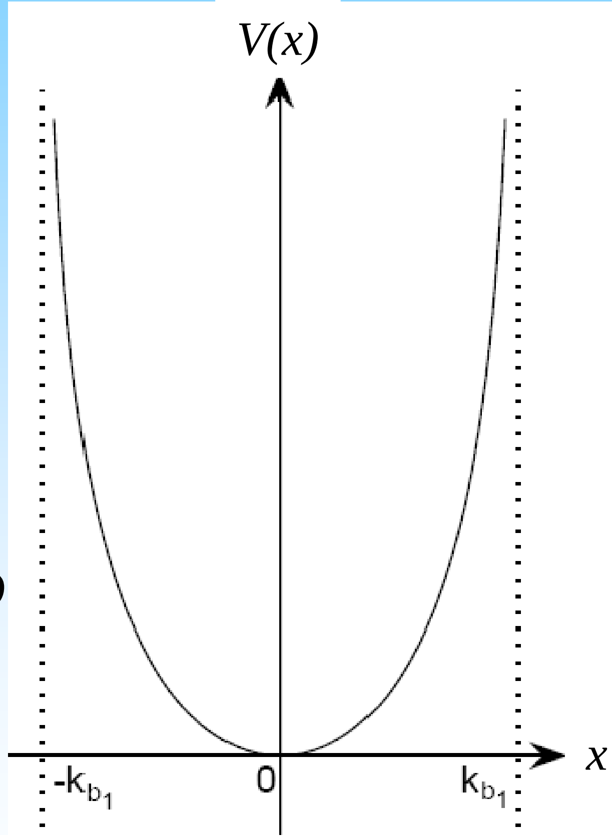
Definition:

A BLF is a scalar function $V(x)$, defined with respect to

$$\dot{x} = f(x) \ , \qquad x(0) \in D \subset R^n \qquad (1)$$

on an open region D containing the origin, with the properties:

- continuous, positive definite, and continuously differentiable at every point of D
- $V(x) \to \infty$ as x approaches the boundary of D, and
- $V(x(t)) \leq b \ \ \forall t > 0$ along the solution of (1), where b is a positive constant.



Technical Lemma

Suppose that there exist continuously differentiable positive definite functions $U: R^l \to R_+$ and $V_i: Z_i \to R_+$, $i = 1,\dots,n$, such that

$$V_i(z_i) \to \infty \quad \text{as} \quad z_i \to -k_{a_i} \text{ or } z_i \to k_{b_i}$$

$$\gamma_1(\|w\|) \le U(w) \le \gamma_2(\|w\|)$$

where γ_1, γ_2 are class K_∞ functions. Let $V = \sum_{i=1}^{n} V_i(z_i) + U(w)$.

For initial conditions $z_i(0)$ starting from $z_i \in (-k_{a_i}, k_{b_i})$, $i = 1,\dots,n$, if the following inequality holds

$$\dot{V} = \frac{\partial V}{\partial z} h \le 0$$

then $z_i(t)$ remains in the open set $z_i \in (-k_{a_i}, k_{b_i})$, $\forall t \in [0, \infty)$.

Technical Lemma

Proof Logic:

i) From conditions on h, existence and uniqueness of solution $z(t)$ is ensured on $t \in [0, \tau_{\max})$, based on (Sontag, 1998).

ii) From $\dot{V} \leq 0$, we know that each $V_i(z_i(t))$ is bounded for $t \in [0, \tau_{\max})$, which implies that

$$-k_{a_i} < z_i(t) < k_{b_i} \quad \text{for } t \in [0, \tau_{\max}).$$

iii) Finally, show that $\tau_{\max} = \infty$. From the boundedness of $V(\eta(t))$, we can show that $\eta(t)$ belongs to a compact set for $t \in [0, \tau_{\max})$. Based on (Sontag, 1998), $\eta(t)$ is defined for all $t \in [0, \infty)$. Therefore, $-k_{a_i} < z_i(t) < k_{b_i}$ for $t \in [0, \infty)$.

E. Sontag, *Mathematical Control Theory. Second edition*. Springer-Verlag, 1998.

Control of Output-Constrained Nonlinear Systems

Output Constraint Problem

Consider nonlinear systems in strict feedback form

$$\dot{x}_i = \theta^T \psi_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}, \qquad i = 1, 2, \ldots, n-1$$

$$\dot{x}_n = \theta^T \psi_n(\bar{x}_n) + g_n(\bar{x}_n)u$$

$$y = x_1$$

where

$\bar{x}_i := [x_1, x_2, \ldots, x_i]^T \in R^i$ are the states

θ is a vector of uncertain parameters

ψ_i and g_i are known smooth functions

$u \in R$ is the input, $y \in R$ is the output

Control Objectives:

1. Track desired trajectory $y_d(t)$
2. Ensure output constraint $|y| < k_{c_1}$ is met for all time
3. All closed loop signals bounded

Assumptions

A.1.1. There exist positive constants $\underline{Y}_0, \bar{Y}_0, A_0, Y_1, \ldots, Y_n$ such that

$$\max\{\underline{Y}_0, \bar{Y}_0\} \le \mathrm{A}_0 < k_{c_1}$$

$$-\underline{Y}_0 \le y_d(t) \le \bar{Y}_0$$

$$|\dot{y}_d(t)| \le Y_1, \quad \ldots, \quad |y_d^{(n)}(t)| \le Y_n$$

A.1.2. The control gain functions $g_i(\cdot),\ i = 1, 2, \ldots, n,$ are known, and there exists a positive constant g_0 such that

$$0 < g_0 \le |g_i(\cdot)|, \qquad i = 1, 2, \ldots, n$$

Without loss of generality, we further assume that $g_i(\cdot)$ are positive.

Motivating Example: Known Second Order System

Consider the system:

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2$$
$$\dot{x}_2 = f_2(x_1, x_2) + g_1(x_1, x_2)u$$

<u>Step 1:</u> Denote $z_1 := x_1 - y_d$.

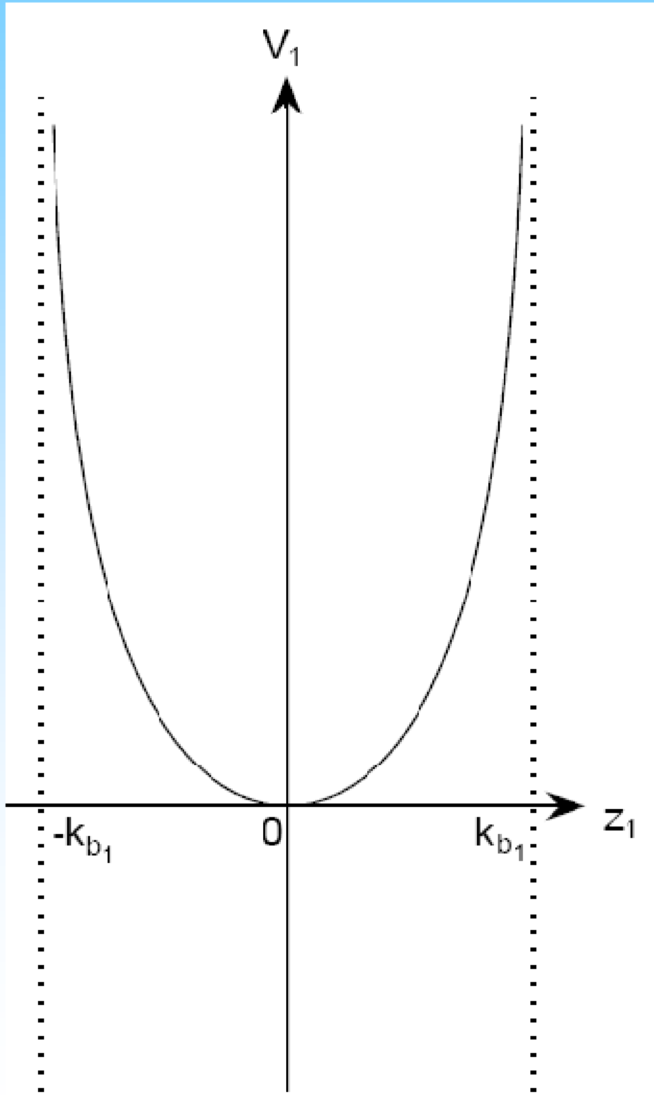
Barrier Lyapunov Function candidate:

$$V_1 = \frac{1}{2}\log\frac{k_{b_1}^2}{k_{b_1}^2 - z_1^2}$$

Design stabilizing function:

$$\alpha_1 = \frac{1}{g_1}\left(-f_1 - \left(k_{b_1}^2 - z_1^2\right)\kappa_1 z_1 + \dot{y}_d\right)$$

$$\rightarrow \dot{V}_1 = -\kappa_1 z_1^2 + \frac{g_1 z_1 z_2}{k_{b_1}^2 - z_1^2}$$



Motivating Example: Known Second Order System

Step 2:

Lyapunov Function candidate:

$$V_2 = V_1 + \frac{1}{2} z_2^2$$

repels z_1 away from barrier

Control law:

$$u = \frac{1}{g_2}\left(-f_2 + \dot{\alpha}_1 - \kappa_2 z_2 - \frac{g_1 z_1}{k_{b_1}^2 - z_1^2} \right)$$

Derivative of V_2 along closed loop trajectories:

$$\dot{V}_2 = -\sum_{i=1}^{2} \kappa_i z_i^2$$

Motivating Example: Known Second Order System

Closed loop error dynamics:

$$\dot{z}_1 = -\left(k_{b_1}^2 - z_1^2\right)\kappa_1 z_1 + g_1 z_2$$
$$\dot{z}_2 = -\kappa_2 z_2 - \frac{g_1 z_1}{k_{b_1}^2 - z_1^2}$$

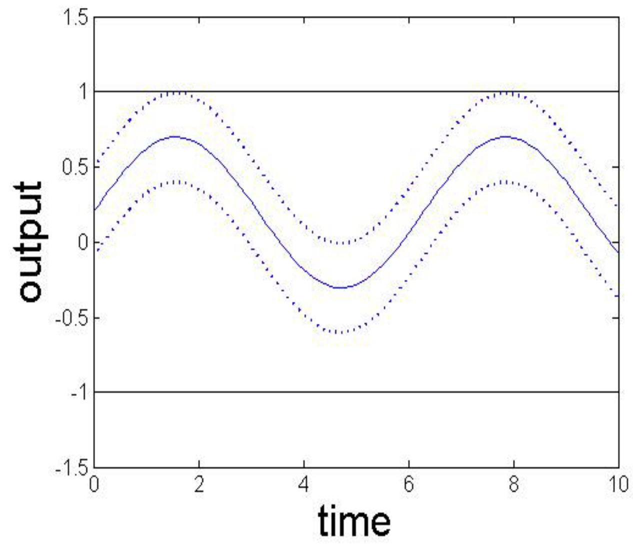
R.H.S. is locally Lipschitz in the set $|z_1| < k_{b1}$

Together with $\dot{V}_2 \le 0$ and $|z_1(0)| < k_{b1}$, we can invoke the Technical Lemma to show

$$| z_1(t) | < k_{b_1} \quad \text{for } t \in [0, \infty)$$

Transform bounds to x_1 coordinates:

$$\begin{aligned} | x_1(t) | &\le | y_d(t) | + | z_1(t) | \\ &\le A_0 + k_{b_1} \\ &< k_{c_1} \quad \text{for } t \in [0, \infty) \end{aligned}$$



Adaptive Control Design

Adaptive Backstepping

Step 1: Barrier Lyapunov function candidate

$$V_1 = \frac{1}{2}\log\frac{k_{b_1}^2}{k_{b_1}^2 - z_1^2} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}, \qquad \text{where } k_{b_1} = k_{c_1} - A_0$$

Step 2 to n: Quadratic Lyapunov function candidates

$$V_i = V_{i-1} + \frac{1}{2}z_i^2, \quad i = 2,\ldots,n$$

Design stabilizing functions α_i , i=1,…,n-1, control and adaptation laws to obtain

$$\dot{V}_n \leq 0$$

Constraint Satisfaction

Invoke Technical Lemma to show that $|z_1(t)| < k_{b1}$

From Assumption 1.1, it follows that $|x_1(t)| < k_{c1}$

Main Results: Symmetric BLF

Consider the closed loop system under the proposed control.

Given that $z_1(0) < k_{b_1}$, we have the results:

i) The signals $z, \hat{\theta}$ remain in the compact sets:

$$\Omega_z = \left\{ z \in R^n : \; |z_1| \le k_{b_1} \sqrt{1 - e^{-2\bar{V}_n}}, \; \|z_{2:n}\| \le \sqrt{2\bar{V}_n} \right\}$$

$$\Omega_{\hat{\theta}} = \left\{ \hat{\theta} \in R^l : \; |\hat{\theta}| \le \theta_M + \sqrt{\frac{2\bar{V}_n}{\lambda_{\min}(\Gamma^{-1})}} \right\}$$

where

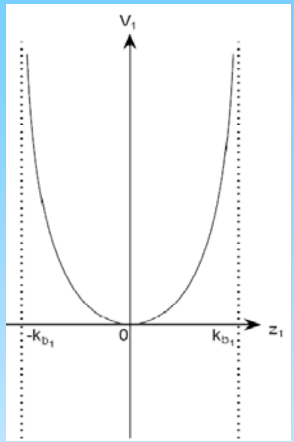
$$\bar{V}_n = \frac{1}{2}\log\frac{k_{b_1}^2}{k_{b_1}^2 - z_1^2(0)} + \frac{1}{2}\sum_{j=2}^{n} z_j^2(0) + \frac{1}{2}\lambda_{\max}(\Gamma^{-1})(\|\hat{\theta}(0)\| + \theta_M)$$

ii) The output is constrained $|y| \le D_{z_1} + A_0 < k_{c_1}, \quad \forall t > 0$

iii) All closed loop signals are bounded

iv) Asymptotic output tracking $y(t) \to y_d(t)$ as $t \to \infty$

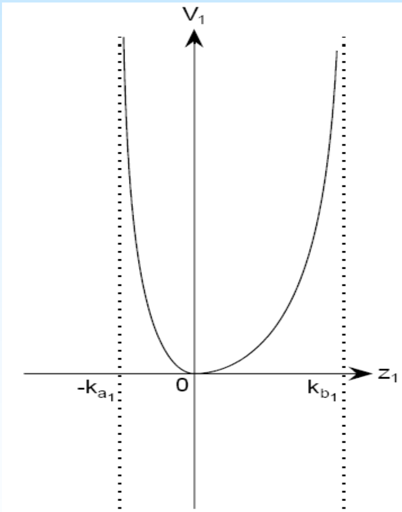
Symmetric vs Asymmetric BLFs



Symmetric

$$V_1 = \frac{1}{2}\log\frac{k_{b_1}^2}{k_{b_1}^2 - z_1^2}$$

yields smooth stabilizing functions α_i, $i=1,\ldots,n$



Asymmetric

$$V_1 = \frac{1}{p}q(z_1)\log\frac{k_{b_1}^p}{k_{b_1}^p - z_1^p} + \frac{1}{p}(1-q(z_1))\log\frac{k_{a_1}^p}{k_{a_1}^p - z_1^p}$$

$$q(\square) = \begin{cases} 1, & \text{if } \square > 0 \\ 0, & \text{if } \square \le 0 \end{cases}$$

To ensure differentiability of α_i, $i=1,\ldots,n$:

- choose even integer $p \ge n$
- careful design of α_1 and α_2

Asymmetric BLF-based Design

Stabilizing functions and control law:

$$\alpha_1 = \frac{1}{g_1}\left[-f_1 - \Big(q(k_{b_1}^p - z_1^p) + (1-q)(k_{a_1}^p - z_1^2)\Big)\kappa_1 z_1^m + \dot{y}_d\right]$$

$$\alpha_2 = \frac{1}{g_2}\left[-f_2 - \kappa_2 z_2 + \dot{\alpha}_1 - \left(\frac{q}{k_{b_1}^p - z_1^p} + \frac{1-q}{k_{a_1}^p - z_1^p}\right) g_1 z_1^{p-1}\right]$$

$$\alpha_i = \frac{1}{g_i}\Big(-f_i - \kappa_i z_i + \dot{\alpha}_{i-1} - g_{i-1} z_{i-1}\Big), \qquad i = 3, \ldots, n$$

$$u = \alpha_n$$

Choosing

- even integer $p \geq n$
- odd integer $m \geq \max\{3, n\}$

ensures that

$\alpha_i(\bar{x}_i, \bar{z}_i, \bar{y}_{d_i})$ is C^{n-i} in the set $z_1 \in \left(-k_{a_i}, k_{b_i}\right)$

Main Results: Asymmetric BLF

Consider the closed loop system under the proposed control.

Given that $-k_{a_1} < z_1(0) < k_{b_1}$, we have the results:

i) The signals $z, \hat{\theta}$ remain in the compact sets:

$$\Omega_z = \left\{ z \in R^n : -\underline{D}_{z_1} \le z_1 \le \bar{D}_{z_1},\ \left\| z_{2:n} \right\| \le \sqrt{2\bar{V}_n} \right\}$$

$$\Omega_{\hat{\theta}} = \left\{ \hat{\theta} \in R^l :\ |\hat{\theta}| \le \theta_M + \sqrt{\frac{2\bar{V}_n}{\lambda_{\min}(\Gamma^{-1})}} \right\}$$

where $\bar{V}_n := \frac{1}{p} q \log \frac{k_{b_1}^p}{k_{b_1}^p - z_1^p(0)} + \frac{1}{p}(1-q)\log \frac{k_{a_1}^p}{k_{a_1}^p - z_1^p(0)} + \frac{1}{2}\sum_{j=2}^{n} z_j^2(0) + \frac{1}{2}\lambda_{\max}(\Gamma^{-1})(\left\|\hat{\theta}(0)\right\| + \theta_M)^2$

$$\underline{D}_{z_1} := k_{b_1}\left(1 - e^{-2\bar{V}_n}\right)^{\frac{1}{p}}$$

$$\bar{D}_{z_1} := k_{a_1}\left(1 - e^{-2\bar{V}_n}\right)^{\frac{1}{p}}$$

ii) All closed loop signals are bounded

iii) The output is constrained $|y(t)| \le D_{z_1} + A_0 < k_{c_1}, \quad \forall t > 0$

iv) Asymptotic output tracking $y(t) \to y_d(t)$ as $t \to \infty$

Comparison with Quadratic Lyapunov Functions

QLF candidates: $V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1$

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i, \quad i = 2, \ldots, n$$

Standard adaptive backstepping (Krstic et al, 1995):

$$\rightarrow \quad \dot{V}_n = -\sum_{i=1}^{n} \kappa_i z_i^2$$

$$\rightarrow \quad z_1^2(t) \le \sum_{i=1}^{n} \left[z_i^2(0) + \lambda_{\max}\left(\Gamma^{-1}\right)\left(\left\| \hat{\theta}_i(0) \right\| + \theta_M \right)^2 \right]$$

Sufficient initial conditions to ensure $|z_1(t)| \le k_{b_1}$:

$$\begin{cases} \left\| \bar{z}_n(0) \right\| \le \sqrt{k_{b_1}^2 - \lambda_{\max}\left(\Gamma^{-1}\right) \sum_{i=1}^{n} \left(\left\| \hat{\theta}_i(0) \right\| + \theta_M \right)^2} \\ k_{b_1}^2 > \lambda_{\max}\left(\Gamma^{-1}\right) \sum_{i=1}^{n} \left(\left\| \hat{\theta}_i(0) \right\| + \theta_M \right)^2 \end{cases}$$

More conservative than that of BLF-based design: $|z_1(0)| \le k_{b_1}$

Numerical Example

Plant: $\dot{x}_1 = \theta_1 x_1^2 + x_2$

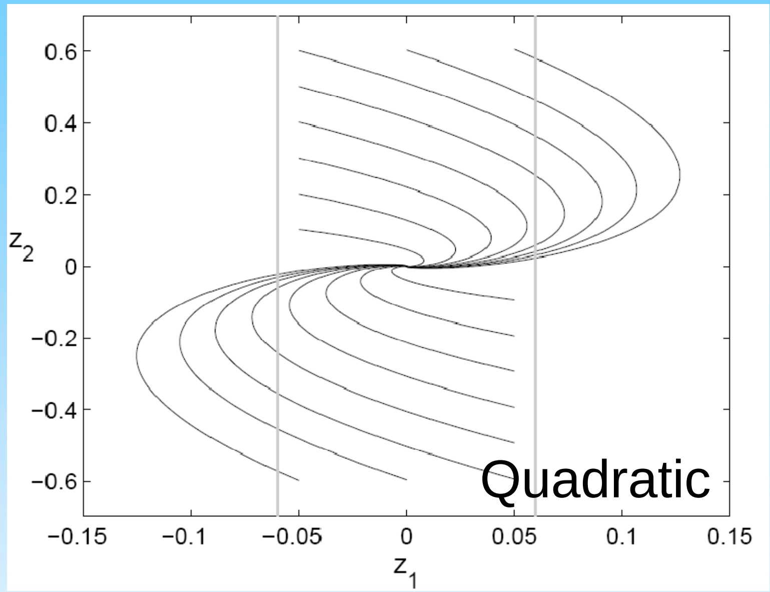
$$\dot{x}_2 = \theta_2 x_1 x_2 + \theta_3 x_1 + (1 + x_1^2)u$$

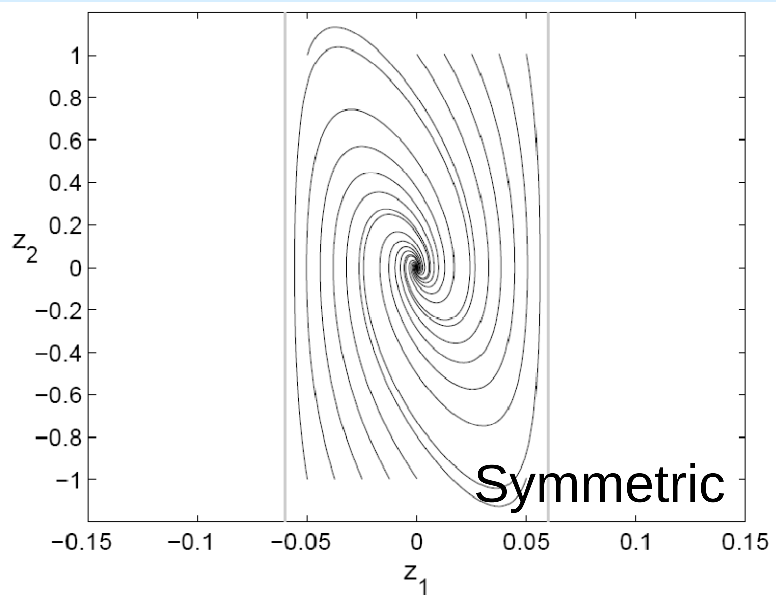
subject to constraint:

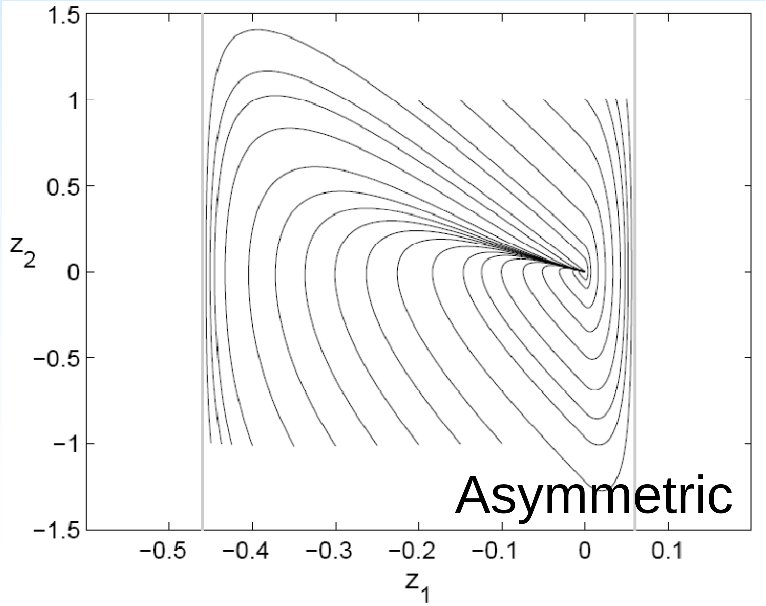
$$|x_1| < 0.56$$

Desired trajectory:

$$y_d(t) = 0.2 + 0.3\sin t$$

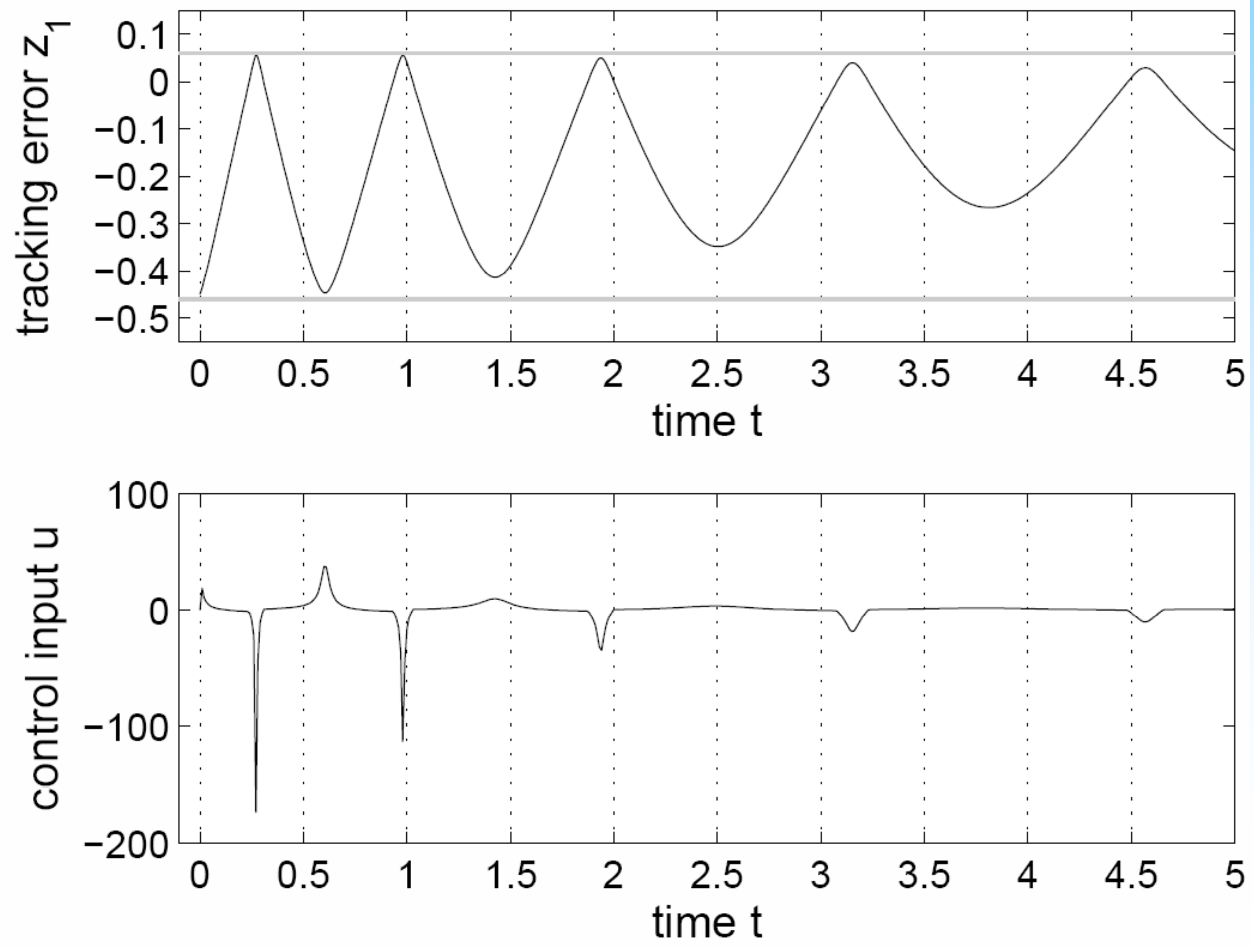






Numerical Example

Control increases rapidly near the barriers to prevent transgression

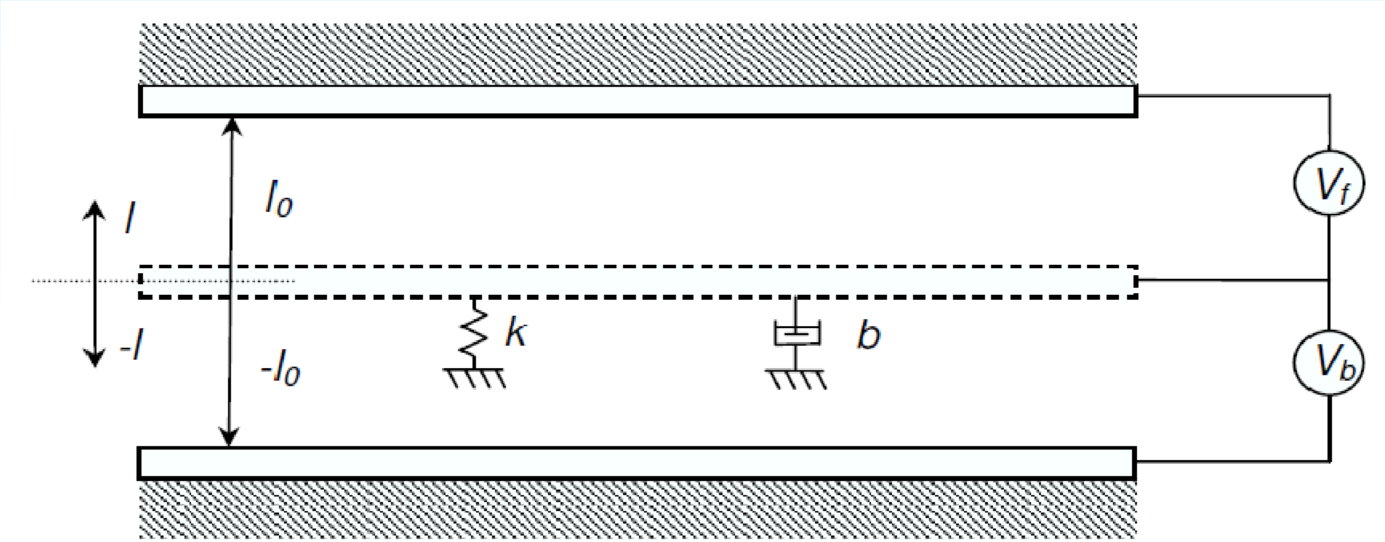


Application Study: Control of Electrostatic Microactuators

Electrostatic Microactuator with Bi-Directional Drive

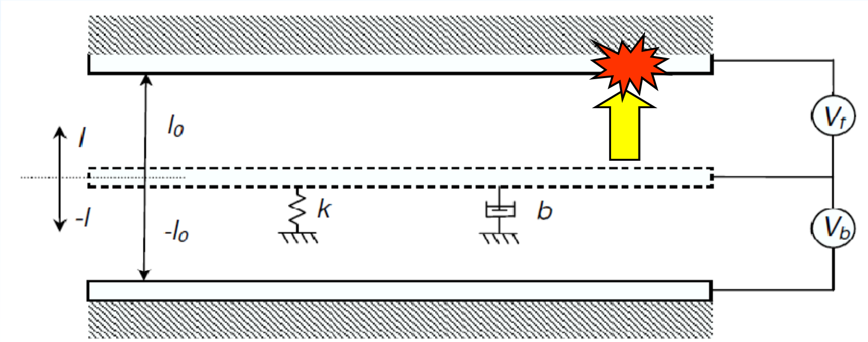
$$m\ddot{l} + b(l)\dot{l} + kl = \frac{\varepsilon A}{2}\left(\frac{V_f^2}{(l_0 - l)^2} - \frac{V_b^2}{(l_0 + l)^2}\right) =: \frac{\varepsilon A}{2}\nu$$

- Parameters unknown
- Track reference trajectory within gap
- Ensure electrodes do not contact (output constraint)



Related Works

- Extending travel range
  - (Chu & Pister, 1994), (Seeger & Crary, 1997), (Chan & Dutton, 2000)
- Nonlinear control of MEMs
  - (Zhu et al, 2005), (Maithripala et al, 2005)
- Adaptive control of MEMs
  - (Shkel et al, 1999), (Park and Horowitz, 2003), (Leland, 2001), (Piyabongkarn et al, 2005)
- Problem of electrode contact is not rigorously treated



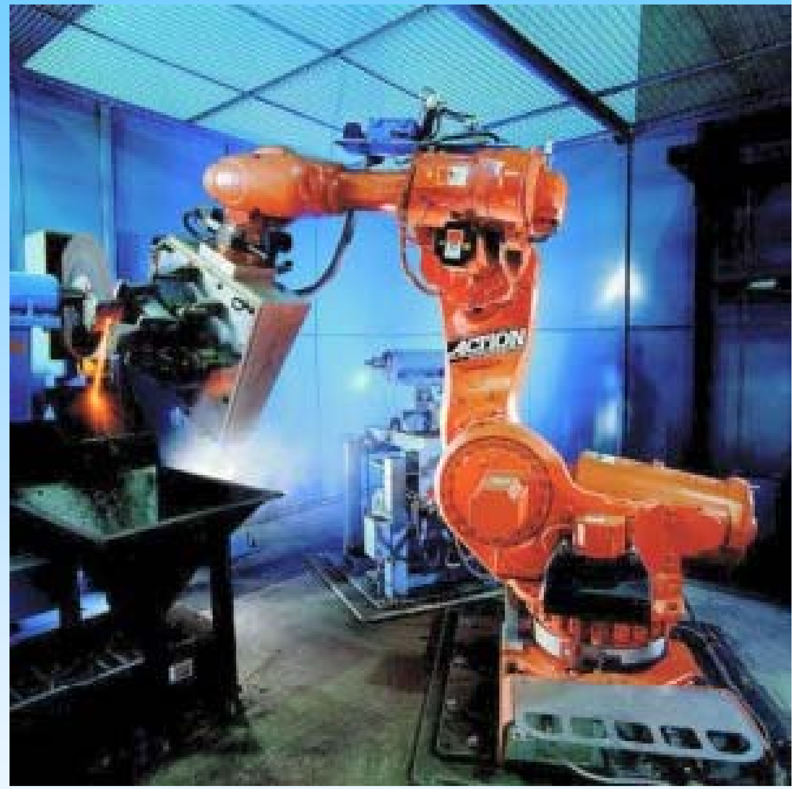
Future Research

NUS National University of Singapore

- Different choices of Barrier Lyapunov Functions
- States Constrained nonlinear systems
- Practical applications

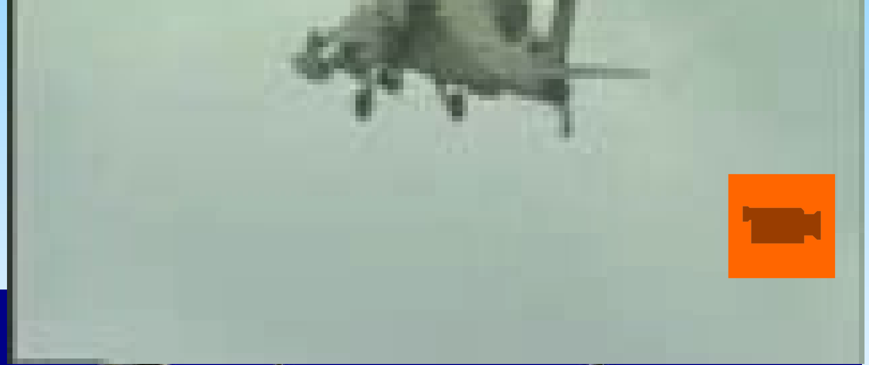
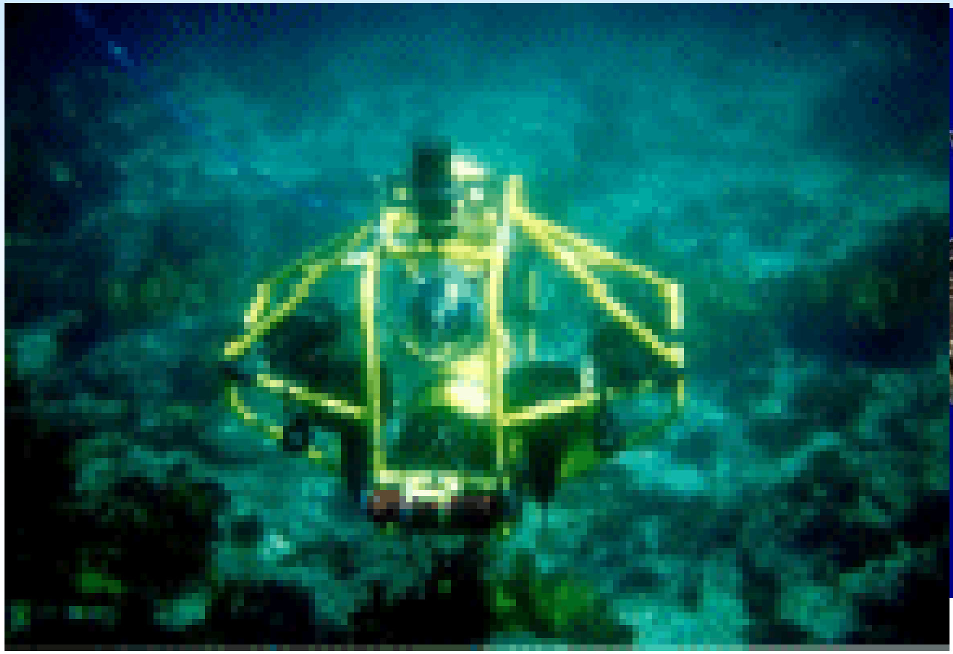
Industrial Robotics

- Rigid, Bulky and Heavy
- Steel Cold



Modern Robotics

- Intelligent autonomous vehicles
- Semi-intelligent autonomous vehicles

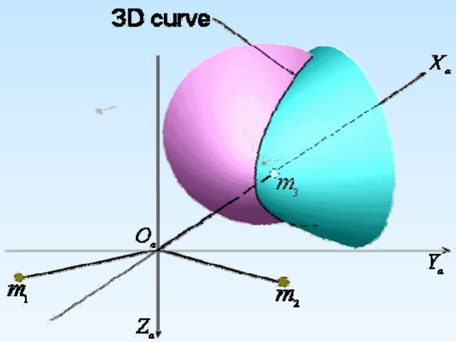


Autonomous Robot X1
Social Robotics Lab
National University of Singapore

Robust Audio Localization

1. Mask Diffraction

2. Multiple Sampling



video

S.S. Ge, A.P. Loh and F. Guan, Robust Sound Localization Using Lower Number of Microphones, *International Journal of Information Acquisition*, 2(1), 2005.

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