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Good Morning

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Stability of Robotic Obstacle Avoidance and Force Interaction

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Previous Research

Haptics & Bilateral Teleoperation

- Colgate & Brown (1994), Miller *et al.* (2004)
- Hannaford & Ryu (2001)
- Diolatti *et al.* (2006), Lee & Li (2003)
- Hokayem & Spong (2006)

Obstacle avoidance & Potentials—A large literature...

- Khatib (1985), Brock and Khatib (1999)

Stability

- Ortega and Spong (1989)
- Johansson (1990a); Johansson (1990b)
- Molander & Willems (1980), Johansson & Robertsson (2002)

Modeling—Preliminaries

Haptics m-DOF master and an m-DOF slave modeled as

$$\begin{aligned} \rho(M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1 + G_1(q_1)) &= \rho(\tau_1 - J_1^T(q_1)F_1) \\ M_2(q_2)\ddot{q}_2 + C_2(q_2,\dot{q}_2)\dot{q}_2 + G_2(q_2) &= \tau_2 - J_2^T(q_2)F_2 \end{aligned} \qquad (1)$$

where

- ρ—a user-specified power-scaling factor;
- (q_1, q_2)—the configuration coordinates;
- (τ_1, τ_2)—the applied forces or control variables;
- (F_1, F_2) are and the environmental forces.

Modeling—Preliminaries

Introduce the potential energy $\mathscr{U}(q)$, kinetic energy $\mathscr{T}(q,\dot{q})$ and the Lagrangian

$$\mathscr{L}(q,\dot{q}) \quad = \quad \mathscr{T}(q,\dot{q}) - \mathscr{U}(q), \tag{2}$$

Equations of motions are provided by the Euler-Lagrange equations (Goldstein 1950)

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} \quad = \quad \tau \tag{3}$$

A configuration-space dynamic model for a serial robot

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) \quad = \quad \tau - J^T(q)F, \quad \tau, q \in \mathbb{R}^n \tag{4}$$

where the position coordinates $q \in \mathbb{R}^n$ with associated velocities $\dot{q} \in \mathbb{R}^n$ are controlled with the applied torques $\tau \in \mathbb{R}^n$; $M(q)$ being the inertia matrix; $C(q,\dot{q})\dot{q}$ the Coriolis and centripetal forces; $G(q)$ gravitation forces and τ the vector of joint torques.

Modeling—Preliminaries

End-effector coordinates x

$$x(t) = f(q), \quad \dot{x} = J(q)\dot{q}, \quad J(q) = \frac{\partial f(q)}{\partial q} \tag{5}$$

Applied forces τ—control torques τ_R, force actuation τ_F, and obstacle avoidance τ_o

$$\tau = \tau_R + \tau_F + \tau_o \tag{6}$$

$$\tau_F = J^T(q)F, \quad x(t) = f(q), \quad \dot{x} = J(q)\dot{q}, \quad J(q) = \frac{\partial f(q)}{\partial q} \tag{7}$$

Obstacle Avoidance

Artificial potential—Khatib (1985)

$$U_o(x) = U_o(f(q)), \quad F_o = -\frac{\partial U_o}{\partial x} \tag{8}$$

where F_o represents a repulsive force created by the artificial potential U_o.

Stability of Trajectory Control

'Computed torque'

$$\tau_R = C(q,\dot{q})\dot{q} + G(q) - M(q)K_V\dot{\tilde{q}} - M(q)K_P\tilde{q} + M(q)\ddot{q}_r \qquad (9)$$

No force interaction

$$M(q)(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) = 0 \qquad (10)$$

$$(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) = 0, \quad M(q) > 0, \forall q \qquad (11)$$

with error dynamics on state-space form

$$\frac{d}{dt}\begin{bmatrix}\dot{\tilde{q}} \\ \tilde{q}\end{bmatrix} = \begin{bmatrix}-K_V & -K_P \\ I & 0\end{bmatrix}\begin{bmatrix}\dot{\tilde{q}} \\ \tilde{q}\end{bmatrix} \qquad (12)$$

Lyapunov function

Introduction of state-space notation for error dynamics

$$\xi = \begin{bmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{bmatrix} \in \mathbb{R}^{2n}, \quad \dot{\xi} = A\xi, \quad A = \begin{bmatrix} -K_V & -K_P \\ I & 0 \end{bmatrix} \qquad (13)$$

A Lyapunov function $V_o(\xi)$ proving stability of the computed torque solutions for unconstrained motion

$$V_o(\xi) = \xi^T P \xi, \quad \frac{dV_o(\xi)}{d\xi} = -\xi^T Q \xi < 0, \quad \|\xi\| \neq 0 \qquad (14)$$

$$-Q = PA + A^T P, \qquad (15)$$

Force Control and Obstacle Avoidance

Stability of Force Control and Obstacle Avoidance??

$$U_o(x) = U_o(f(q)), \quad F_o = -\frac{\partial U_o}{\partial x} \tag{16}$$

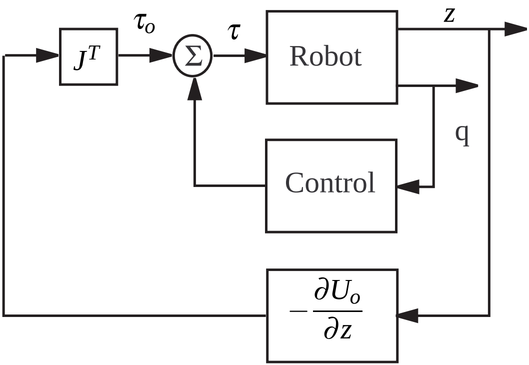
with error dynamics

$$\begin{aligned} M(q)(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) &= \tau_o & (17)\\ (\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) &= M^{-1}(q)\tau_o, \quad M(q) > 0 & (18)\end{aligned}$$

or

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix}\dot{\tilde{q}}\\ \tilde{q}\end{bmatrix} &= \begin{bmatrix}-K_V & -K_P\\ I & 0\end{bmatrix}\begin{bmatrix}\dot{\tilde{q}}\\ \tilde{q}\end{bmatrix} + \begin{bmatrix}M^{-1}(q)\\ 0\end{bmatrix}\tau_o & (19)\\ \tau_o &= J^T F_o = -J^T(q)\frac{\partial U_o}{\partial x} & (20)\end{aligned}$$

Artificial Potential Feedback SPR?



Unless the constrained system is strictly positive real (SPR), the gain margin has a **finite upper limit**, thus restricting the application of artificial potential feedback.

Problem Formulation

Artificial potential field

$$U_o(x) = (x - x_o)^T K_o (x - x_o) \qquad (21)$$

Repulsive force

$$F_o \quad = \quad -\frac{\partial U_o}{\partial x} = -K_o(x - x_o) \qquad (22)$$

Lur'e-Lyapunov function—Popov criterion

$$V(x,z) \quad = \quad x^T P x + 2\eta \int_0^z \psi^T(\zeta)\kappa d\zeta, \quad P > 0,\ \eta > 0 \qquad (23)$$

with

$$\kappa\psi(z) = \frac{\partial U_o}{\partial x} = K_o(x - x_o), z = (x - x_o) = f(q) - f(q_o) \qquad (24)$$

where the second term represents a potential-like term of the Lur'e-Lyapunov function.

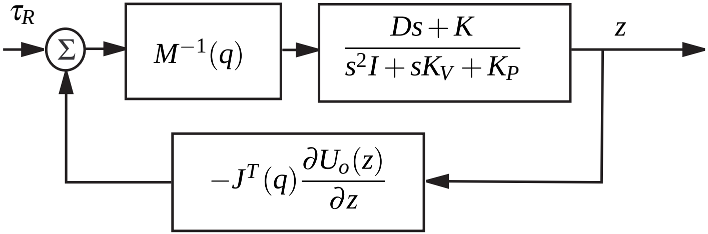
Main Result

Modified artificial potential

$$
\begin{aligned}
U_o(z) &= (z-z_o)^T K_o (z-z_o) && (25)\\
z &= D\dot{\widetilde{q}} + K\widetilde{q} && (26)\\
Z(s) &= (s^2 I + sK_V + K_P)^{-1}(Ds+K) && (27)\\
F_o &= -\frac{\partial U_o(z)}{\partial z} = -K_o(z-z_o) && (28)
\end{aligned}
$$

with D chosen in such a way that the transfer function $Z(s)$ be strictly positive real (SPR)

Modified artificial potential—Impedance variable



SPR Design via Riccati Equation

$$\begin{aligned} 0 &= PA_0 + A_0^T P + Q - PBB^T P, \quad A = A_0 - BB^T P && (29) \\ PA + A^T P &= -Q - PBB^T P, \quad C = B^T P && (30) \\ Q &= Q^T > 0,\ P = P^T > 0 && (31) \end{aligned}$$

Choose

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & 0 \\ I_n & 0 \end{bmatrix}, \quad B = \begin{bmatrix} I_n \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ I_n \end{bmatrix}, && (32) \\ 0 &= PA_0 + A_0^T P + Q - PBB^T P, \quad C = B^T P && (33) \\ D &= K_V = B^T PB, && (34) \\ K &= K_P = B^T PE, && (35) \end{aligned}$$

Lyapunov Function

Introduce the obstacle avoidance acceleration

$$\alpha_o = M^{-1}(q)\tau_o = M^{-1}(q)J^T(q)\frac{\partial U_o(z)}{\partial z} \quad (36)$$

Consider the Lyapunov function (or storage function)

$$V(\xi,z) = \xi^T P\xi + \eta U_o(z), \quad \eta = \text{constant} \quad (37)$$

$$= \xi^T P\xi + \eta \int_0^z \frac{\partial U_o(\zeta)}{\partial \zeta} d\zeta \quad (38)$$

with the derivative (for proof—see Appendix)

$$\frac{dV(\xi,z)}{dt} = \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \mathscr{Q} \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} < 0, \quad \| \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \| \neq 0 \quad (39)$$

$$0 > \mathscr{Q} = \begin{bmatrix} PA + A^T P & PB - \eta A^T C^T K_o^T \\ B^T P - \eta K_o CA & -\eta(K_o CB + B^T C^T K_o^T) \end{bmatrix} \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \quad (40)$$

Design of SPR Obstacle Avoidance

Let the computed-torque open-loop dynamics be decribed by

$$A_0 = \begin{bmatrix} 0 & 0 \\ I_n & 0 \end{bmatrix}, \quad B = \begin{bmatrix} I_n \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ I_n \end{bmatrix}, \tag{41}$$

and solve the Riccati equation

$$0 = PA_0 + A_0^T P + Q - PBB^T P, \quad C = B^T P \tag{42}$$

$$D = K_V = B^T PB, \tag{43}$$

$$K = K_P = B^T PE, \tag{44}$$

Obstacle Avoidance—Example

$$m\ddot{q} = \tau + F, \quad F = 0 \tag{45}$$

$$\tau_R = -mK_V\dot{\widetilde{q}} - mK_P\widetilde{q} + m\ddot{q}_r \tag{46}$$

$$\tau_o = -\frac{\partial U_o(z)}{\partial z} \tag{47}$$

$$\tau = \tau_R + \tau_o \tag{48}$$

with the state-space representation

$$\frac{d\xi}{dt} = \begin{bmatrix} -K_V & -K_P \\ I & 0 \end{bmatrix} \xi + \begin{bmatrix} M^T(q) \\ 0 \end{bmatrix} \tau_o, \ \xi = \begin{bmatrix} \dot{\widetilde{q}} \\ \widetilde{q} \end{bmatrix} \in \mathbb{R}^2 \tag{49}$$

$$z = C\xi = \begin{bmatrix} D & K \end{bmatrix} \xi \tag{50}$$

Obstacle Avoidance—Example (cont'd)

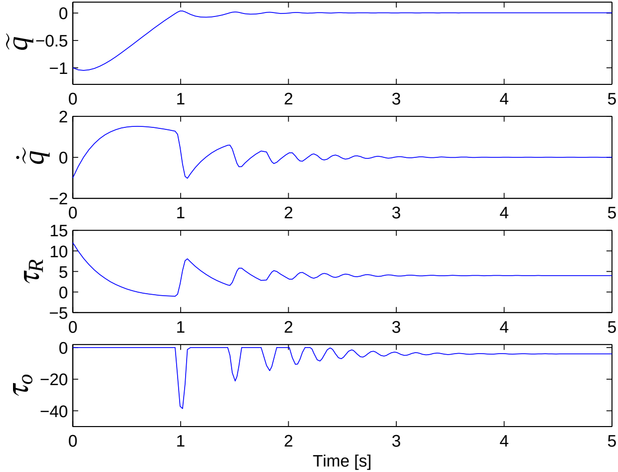
For $m = 1$, $Q = 4I$, $K_o = 10000$, $\eta K_o = 1$, $D = K_V = 5.2362$, $K = K_P = 3.2362$,

$$P = \begin{bmatrix} 5.2360 & 3.2360 \\ 3.2360 & 6.4721 \end{bmatrix} > 0, C = B^T P = \begin{bmatrix} 5.2360 & 3.2360 \end{bmatrix} \quad (51)$$

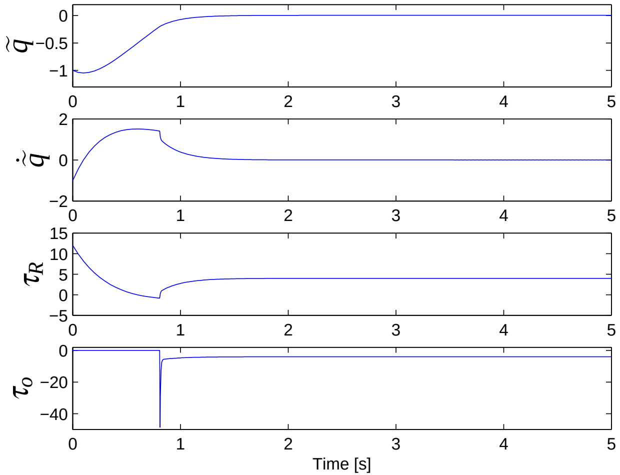
and

$$\begin{aligned} \mathscr{Q} &= \begin{bmatrix} PA + A^T P & PB - A^T C^T \\ B^T P - CA & -(CB + B^T C^T) \end{bmatrix} \quad (52) \\ &= \begin{bmatrix} -48.36 & -27.42 & 2.00 \\ -27.42 & -20.94 & 3.24 \\ 2.00 & 3.24 & -10.47 \end{bmatrix} < 0, \quad \sigma(\mathscr{Q}) = \begin{bmatrix} -65.5143 \\ -10.6925 \\ -3.5690 \end{bmatrix} \end{aligned}$$

Obstacle repulsion $D = 0$

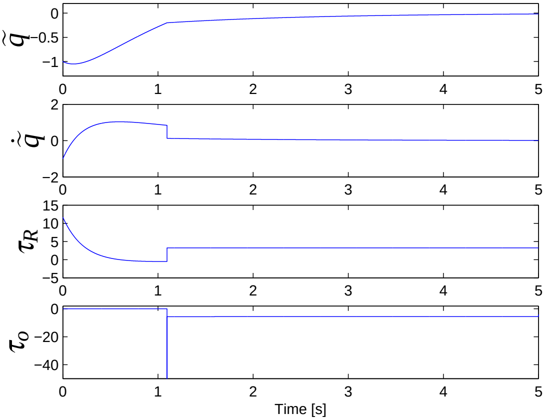


Obstacle repulsion $D = 0.2$



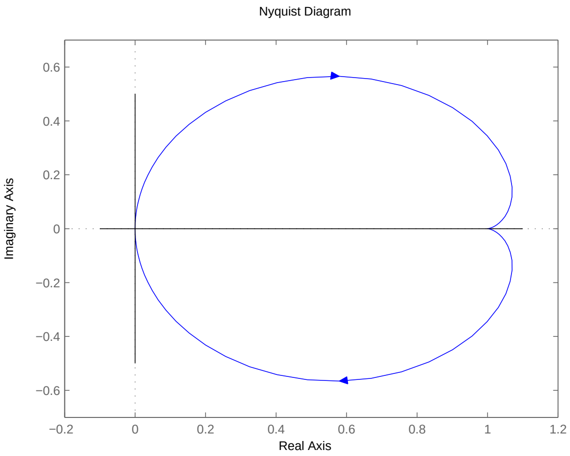
Obstacle repulsion—SPR design

$D = 5.2362$



$K_o = 10000$ and $D = 5.2362$, $K = 3.2362$

Nyquist



Nyquist Diagram
Imaginary Axis
Real Axis
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.2
0
0.2
0.4
0.6
0.8
1
1.2

Conclusions

- Modified artificial potential functions;
- Impedance variable replaces a position variable;
- Block decomposition into a strict positive real (SPR) block and a passive high-gain block;
- Asymptotic stability;
- Elimination of oscillatory or divergent behavior;
- Strictly positive real (SPR) design.

谢谢

Thanks

Tack

Acknowledgements

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Appendix—Stability

Let the obstacle avoidance acceleration be

$$\alpha(z,q) = M^{-1}(q)J^T(q)\frac{\partial U_o(z)}{\partial z} \qquad (53)$$

The closed-loop system represented for $\tau_R = \tau_F = 0$

$$\dot{x} = Ax - B\alpha_o(z,q), \quad B = \begin{bmatrix} I_n \\ 0 \end{bmatrix} \qquad (54)$$

$$z = Cx = \begin{bmatrix} D & K \end{bmatrix} x, \quad CB = D \qquad (55)$$

Appendix—Stability

According to the conditions of the Popov criterion (Popov 1961; Khalil 1996; Johansson and Robertsson 2006), the closed-loop system dynamics with feedback interconnection are described by

$$\begin{bmatrix} \dot{x} \\ -\dot{z} \end{bmatrix} = \begin{bmatrix} A & B \\ -CA & -CB \end{bmatrix} \begin{bmatrix} x \\ -\alpha_o(z) \end{bmatrix}, \; -\alpha_o^T(z)(\alpha_o(z) - \kappa z) \geq 0 \tag{56}$$

Appendix—Stability

The Lyapunov function candidate V and its derivative are

$$V = V(x,z) = x^T Px + 2\eta \int_0^z \alpha_o^T(\zeta)\kappa d\zeta, \quad (57)$$

$$\eta \geq 0, \quad \kappa > 0, \quad \kappa \in \mathbb{R}^{m\times m} \quad (58)$$

$$\frac{dV}{dt} = x^T P\dot{x} + \dot{x}^T Px + 2\eta \alpha_o^T(z)\kappa \dot{z} \quad (59)$$

$$= x^T(PA + A^T P)x - 2x^T PB\alpha_o(z) \quad (60)$$

$$+ \ 2\eta \alpha_o^T(z)\kappa C(Ax - B\alpha_o(z)) \quad (61)$$

$$\frac{dV}{dt} = \begin{bmatrix} x \\ -\alpha_o(z) \end{bmatrix}^T \mathscr{Q} \begin{bmatrix} x \\ -\alpha_o(z) \end{bmatrix} \quad (62)$$

$$\mathscr{Q} = \begin{bmatrix} PA + A^T P & PB - \eta A^T C^T \kappa^T \\ B^T P - \eta \kappa CA & -\eta(\kappa CB + B^T C^T \kappa^T) \end{bmatrix} < 0 \quad (63)$$

Appendix—Stability

In summary, for

$$\mathscr{A}_P = \begin{bmatrix} A & B \\ -\eta\kappa CA - \kappa C & -\eta\kappa CB - I_m \end{bmatrix} \tag{64}$$

a sufficient condition for asymptotic stability is the existence of a solution to the Lyapunov equation

$$\mathscr{P}_o \mathscr{A}_P + \mathscr{A}_P^T \mathscr{P}_o = -\mathscr{Q} \leq 0, \tag{65}$$

and

$$\mathscr{P}_o = \begin{bmatrix} P & 0 \\ 0 & I_m \end{bmatrix} > 0, \quad \mathscr{Q} = \begin{bmatrix} \mathscr{Q}_{11} & \mathscr{Q}_{12} \\ \mathscr{Q}_{12}^T & \mathscr{Q}_{22} \end{bmatrix} \geq 0, \quad \mathscr{Q}_{11} > 0 \tag{66}$$

which renders $dV/dt < 0$ for $\|x\| \neq 0$ and guarantees existence of a Lyapunov function V.