Identification of Monotone Wiener Systems

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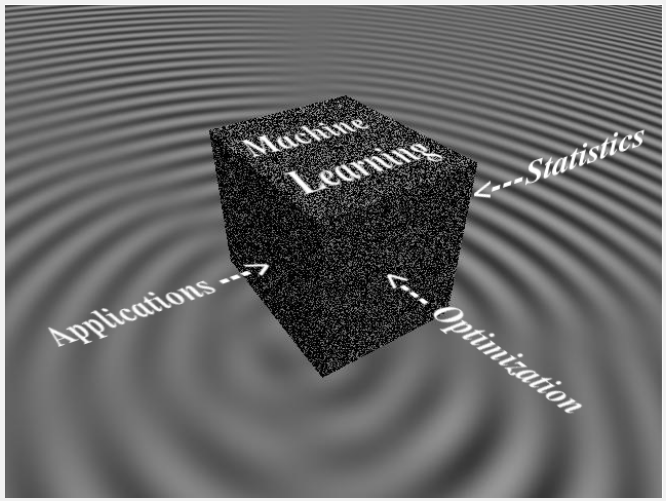
UU/IT, SysCon (Liang Dai) + ESAT/KULeuven/Belgium (Van Belle, Suykens, Van Huffel)

Lund 2011, 5th Swedish-Chinese Conference on Control



Overview

1. Problem Setting - Monotone Wiener Models.
2. Batch Identification - MINLIP.
3. Recursive Identification - RANKTRON.
4. Non-parameteric Control - NORTKNAR.
5. Complexity Control.
6. Conclusions and Outlook.

$\rightarrow$ Specific 2 General.

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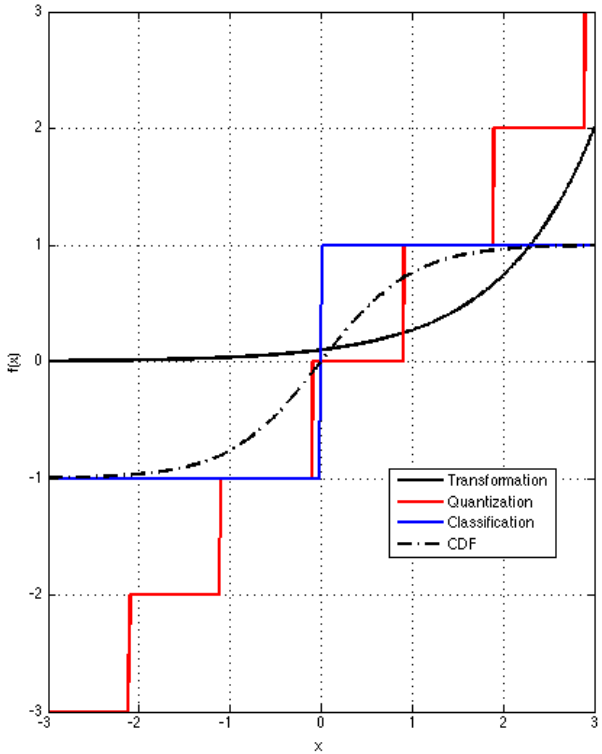

$$y_t = f_0\left(H_0(q^{-1})u_t\right) = f_0\left(\sum_{\tau=0}^{\infty} h_{0,\tau}\, u_{t-\delta}\right),\ \forall t = 1..n$$

- Identification: given $\{(u_t, y_t)\}_{t=1}^{n}$, recover unknown but fixed $(f_0, \mathbf{h}_0)$.
- Monotone: f_0 is monotone in/decreasing.
  - Saturation, e.g.
    $$y_t = f_0(z_t) = \min(z_t, 1).$$
  - Transformed, e.g.
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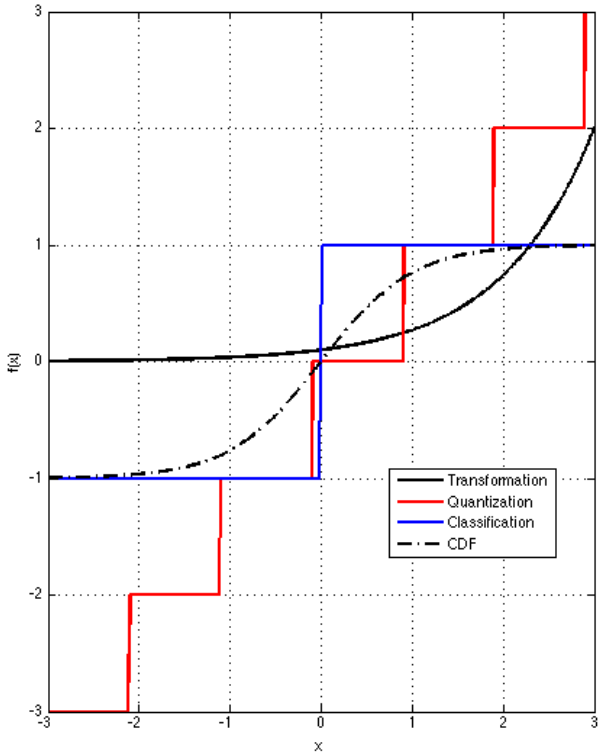

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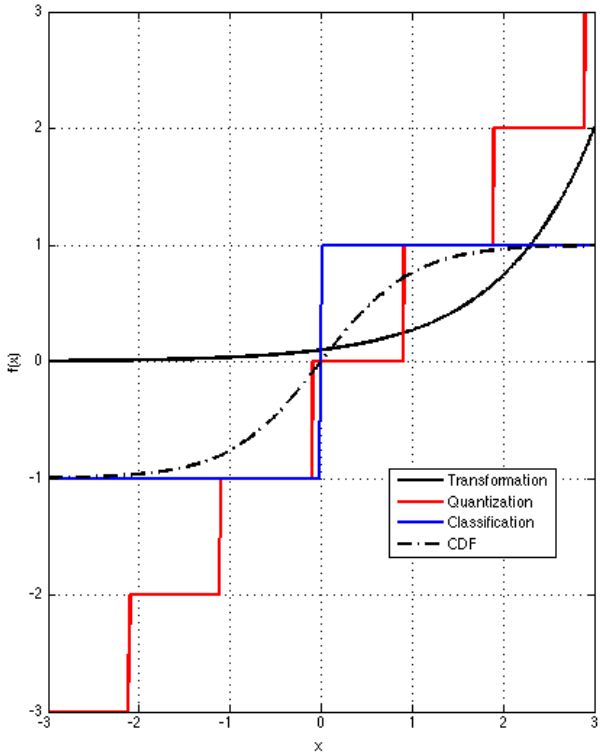

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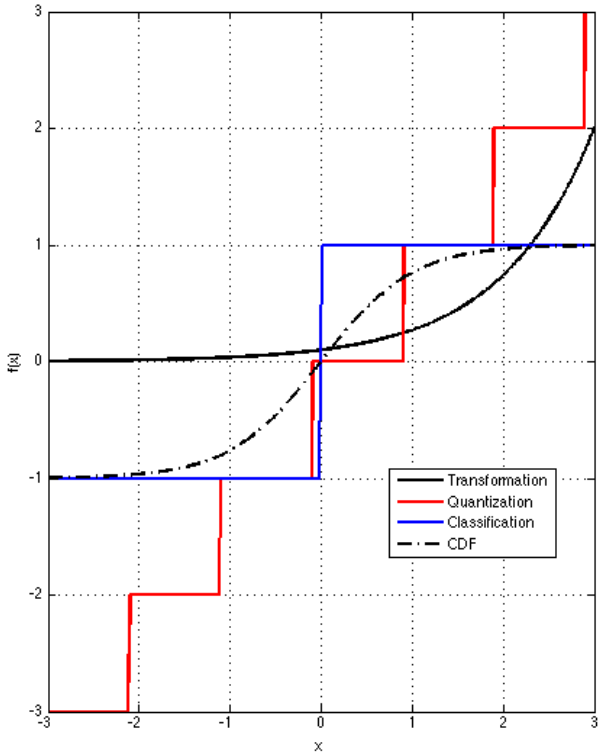

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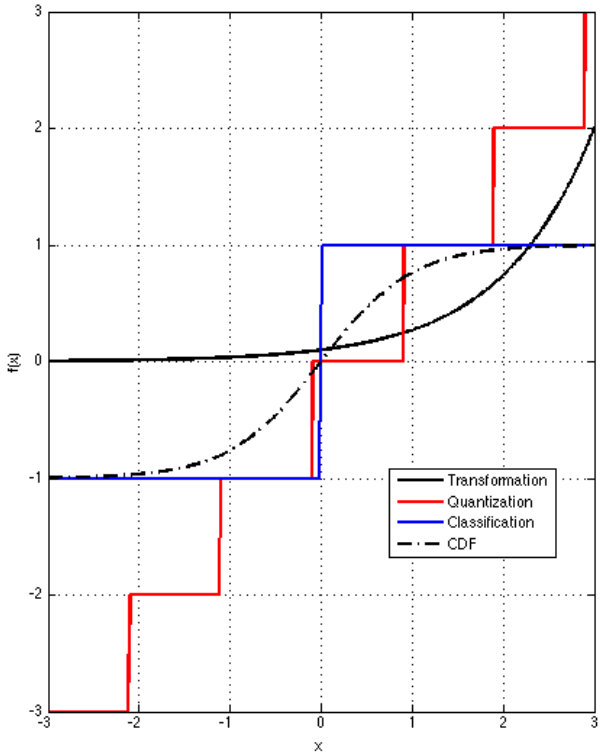

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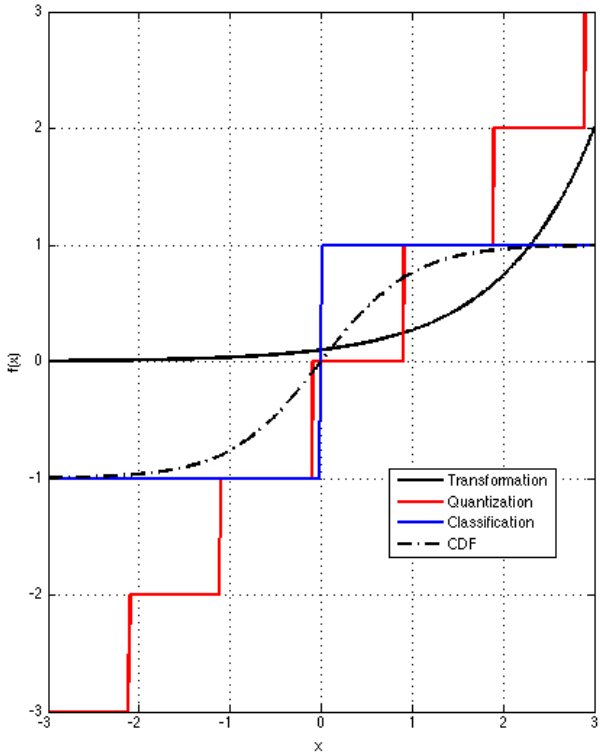

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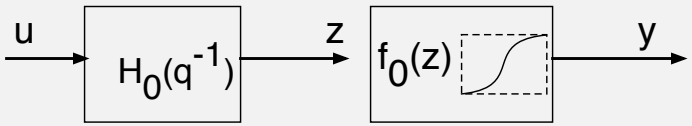

- Noise
- Identifiability.
- Linear-Nonlinear.
- No stochastic assumptions, non-differentiable, discontinuous functions, ...

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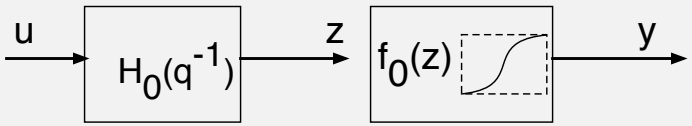

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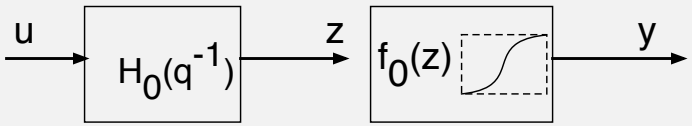

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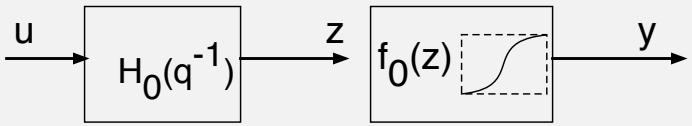

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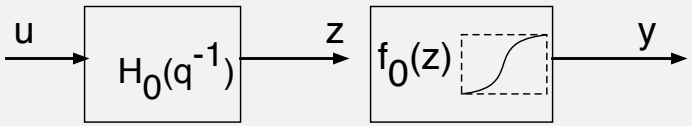

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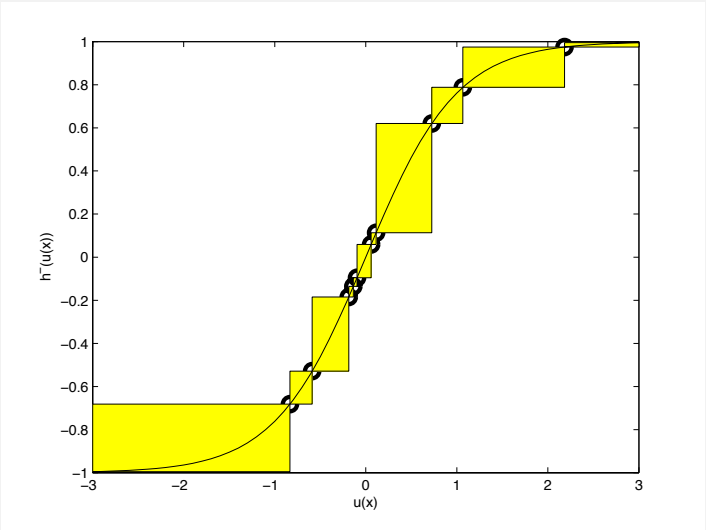

- Preserve order ('monotone')
- Least Complex.
- If too simple a solution $\Rightarrow$ counterexample.
- Convex.
- Cfr.:
  - Regularization - two equally powerful solutions $\rightarrow$ numerical problems.
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  - SVM: Least Complex without classification error.

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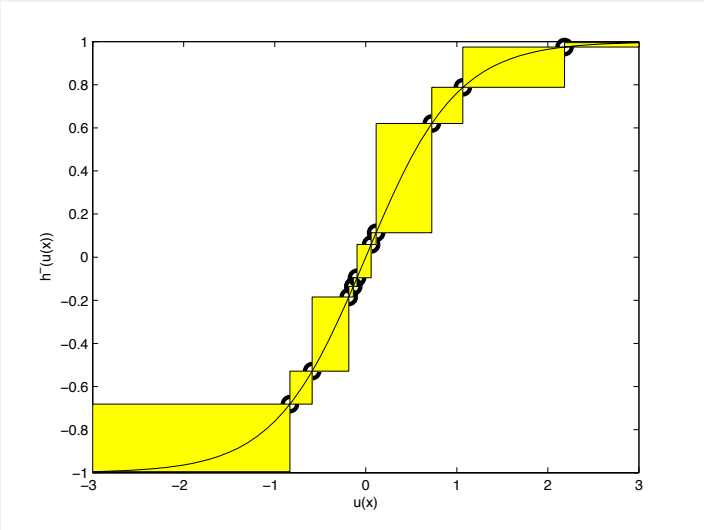

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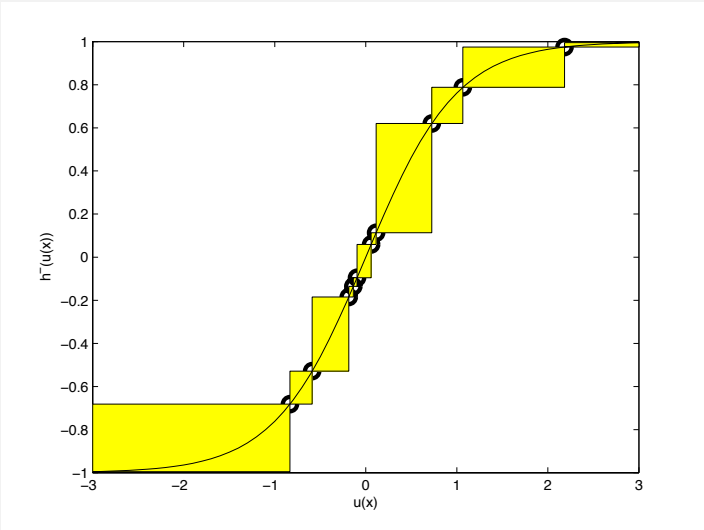

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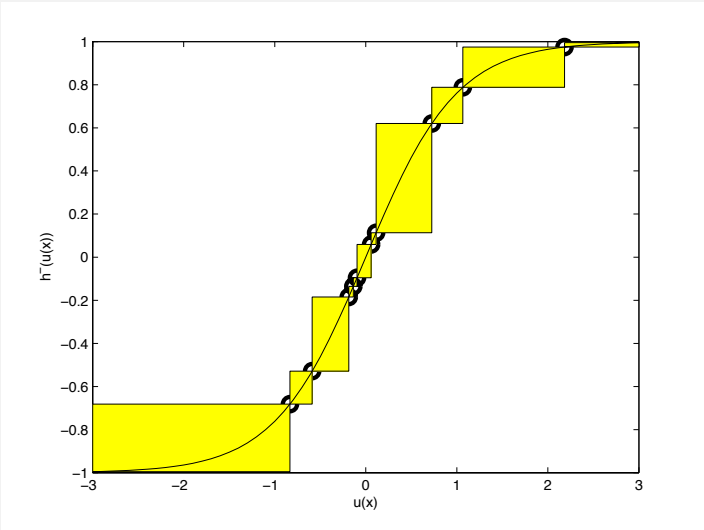

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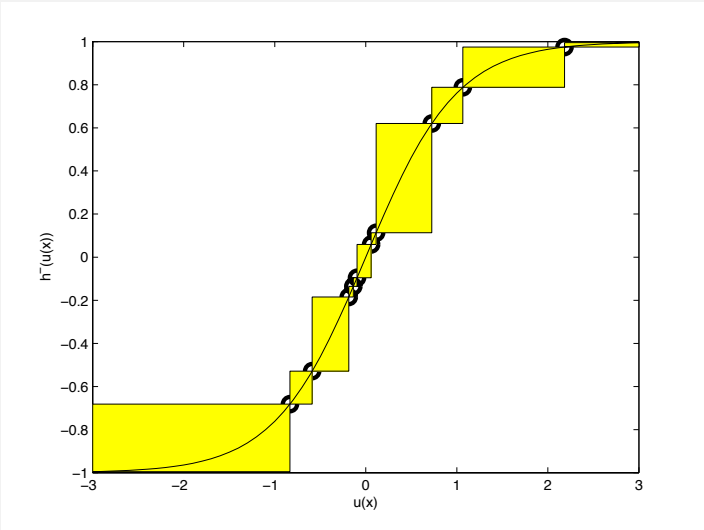

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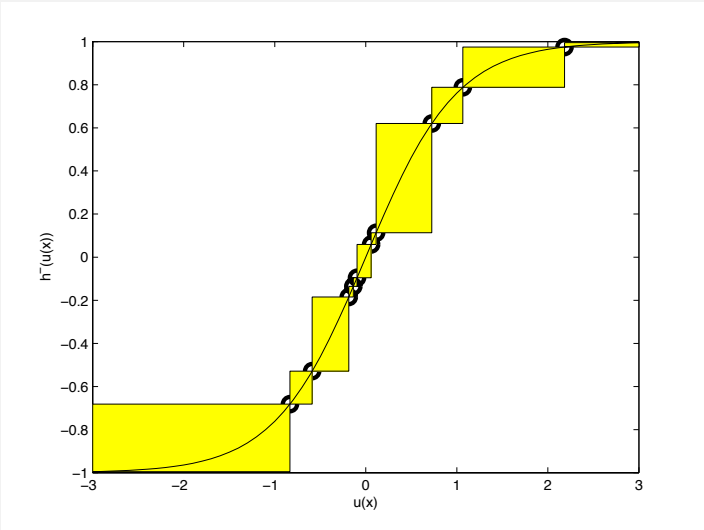

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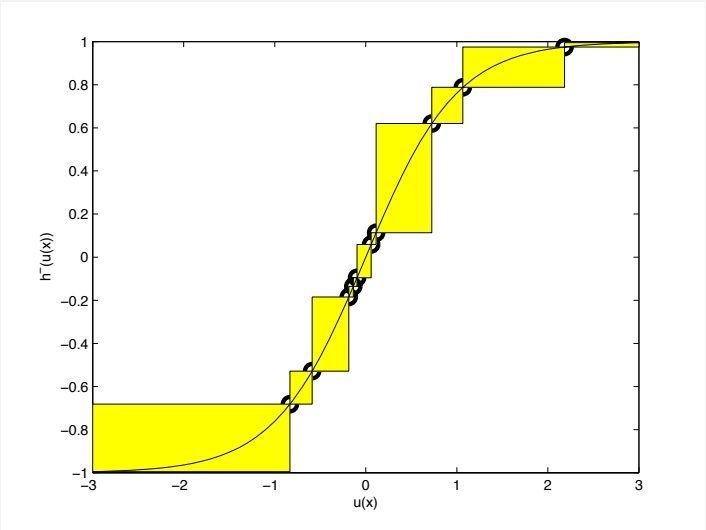

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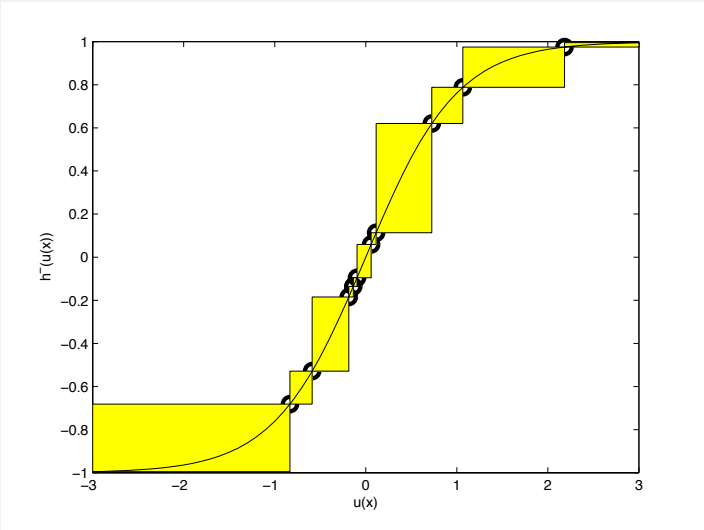

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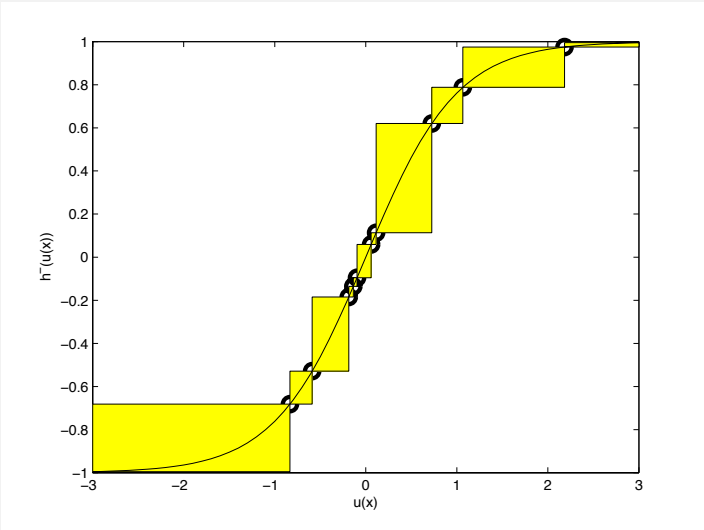

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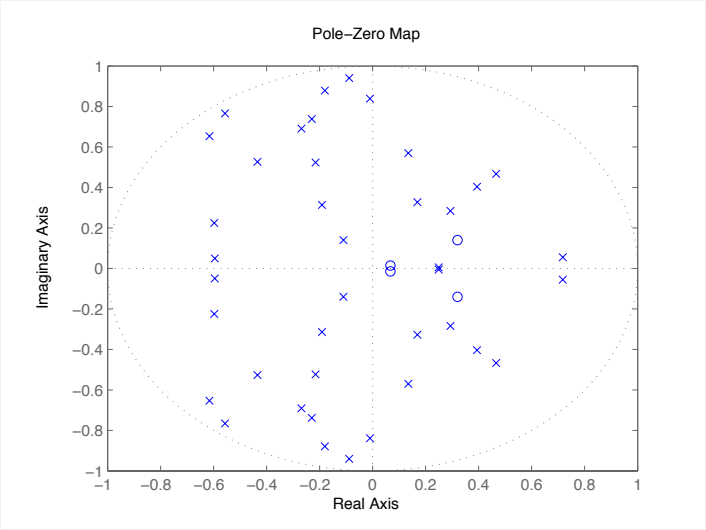

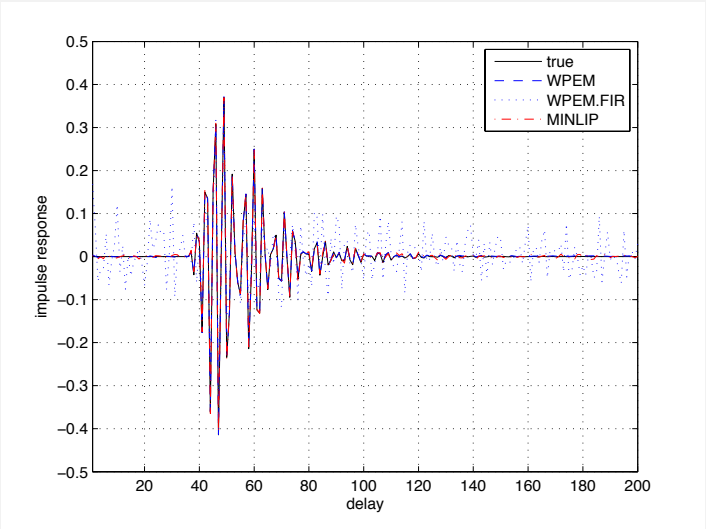

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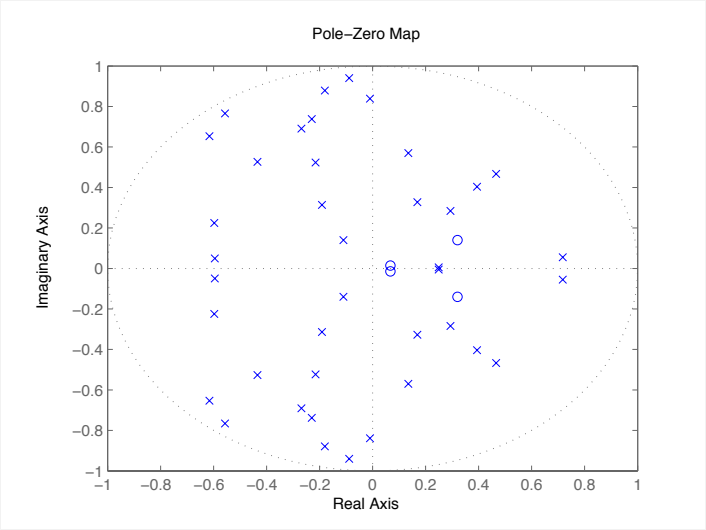

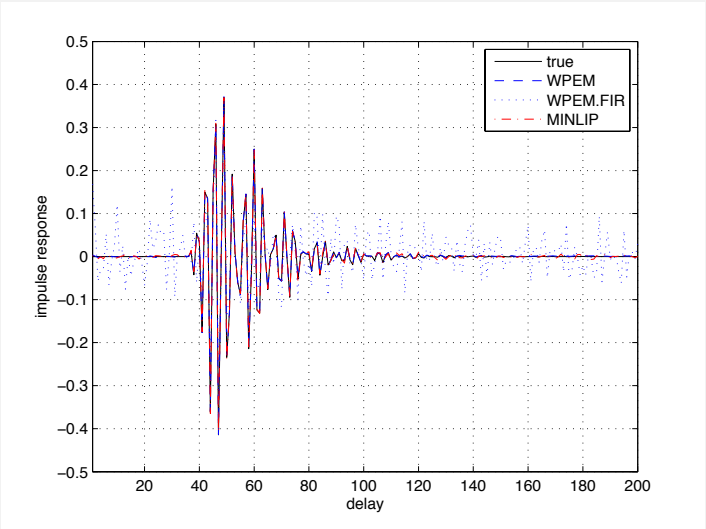

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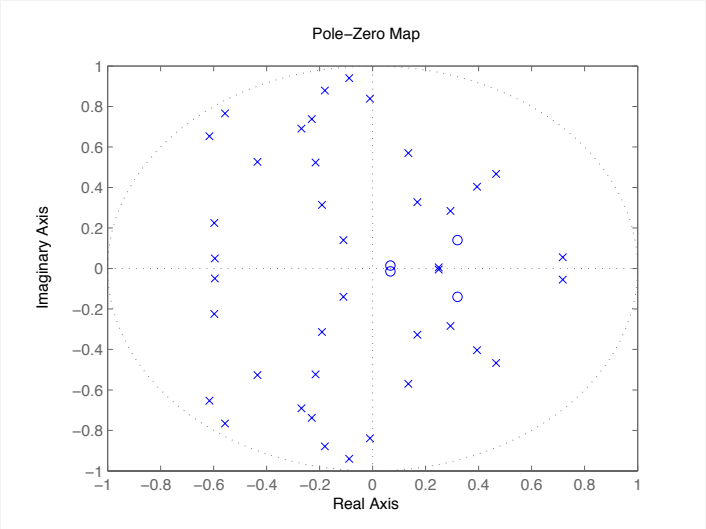

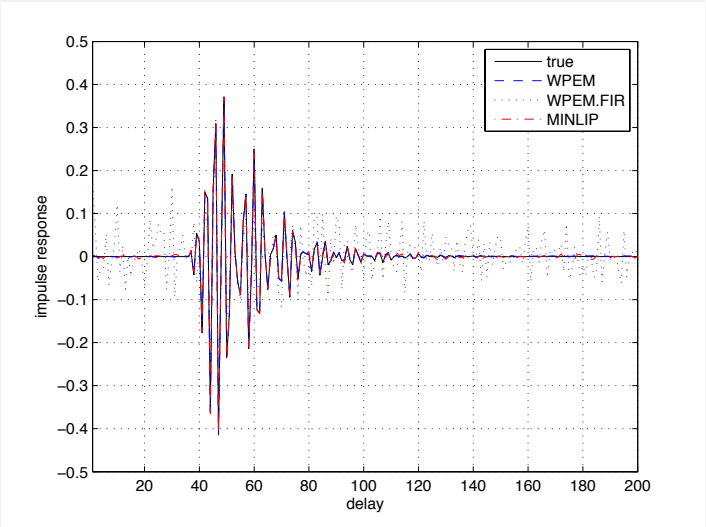

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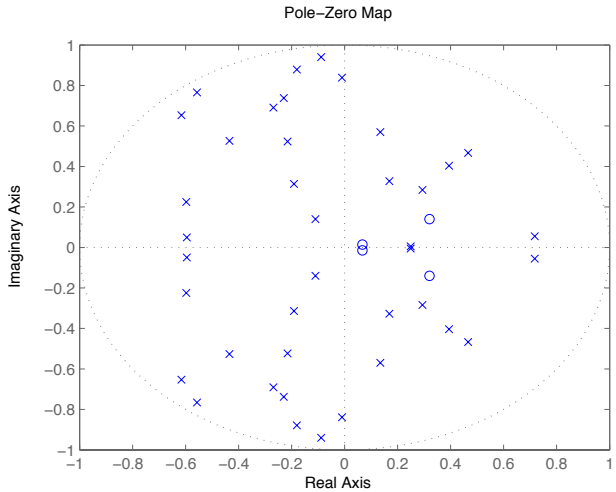

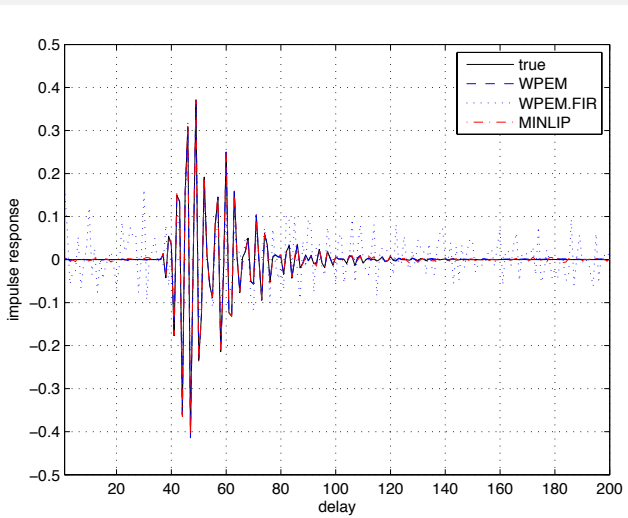

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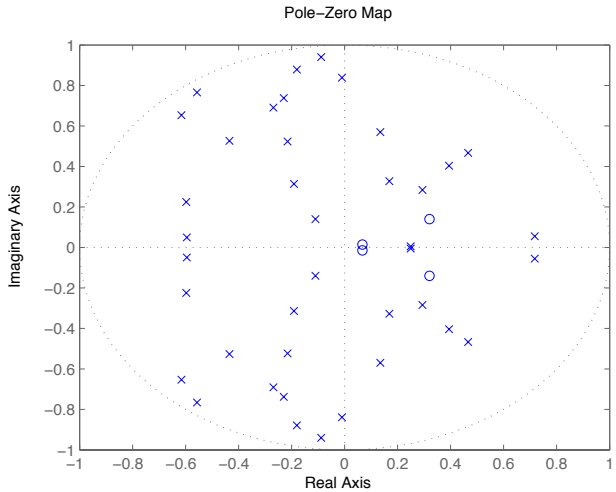
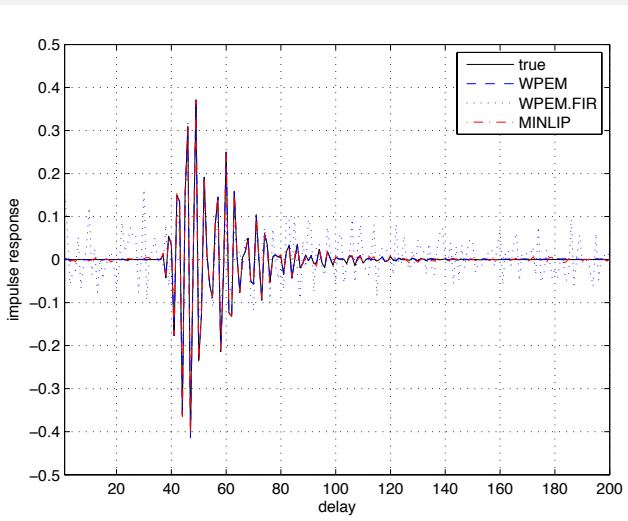
- Almost Consistency (Lipschitz).
- Extensions to Noise.
- Quantization (Margin).

Batch Setting - MINLIP



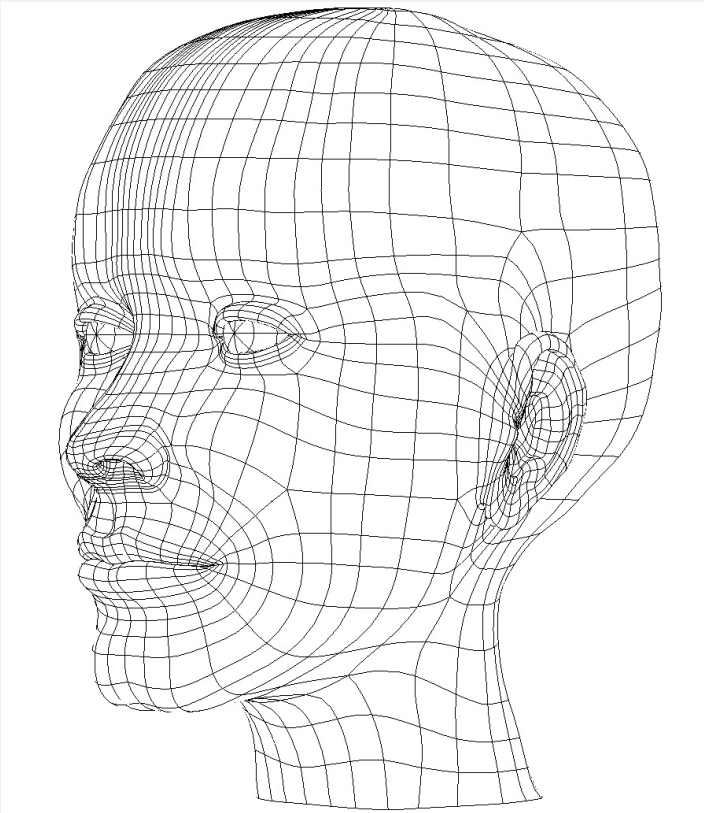


Definition (Local PE)

For every $\mathbf{u}$ one has m linear independent vectors $\{\mathbf{u}_1, \ldots, \mathbf{u}_m\}$ where

$$\|\mathbf{u} - \mathbf{u}_i\|_2 \leq \epsilon$$

Recursive Identification - RANKTRON



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Cfr. PERCEPTRON (1950s)

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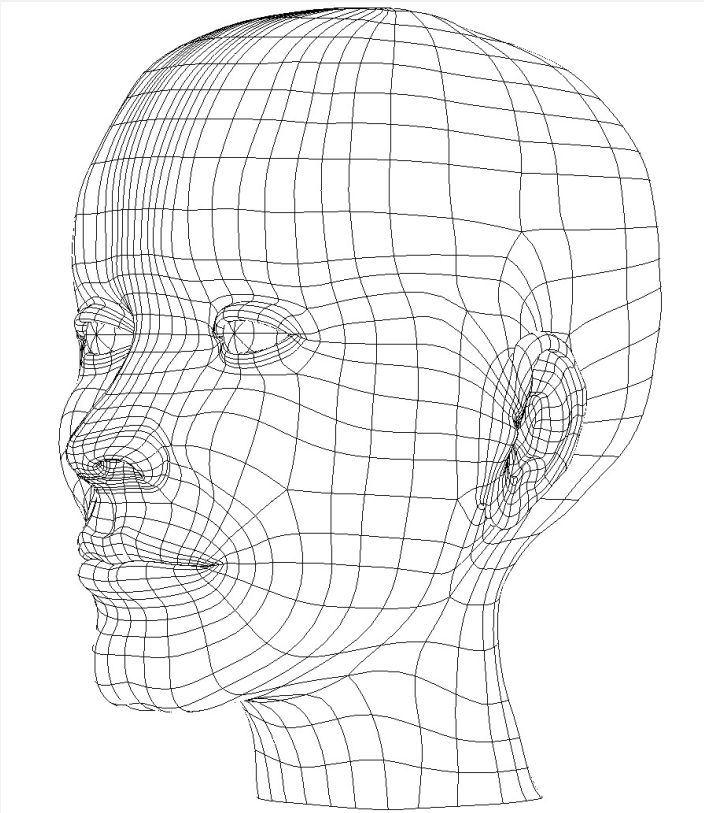

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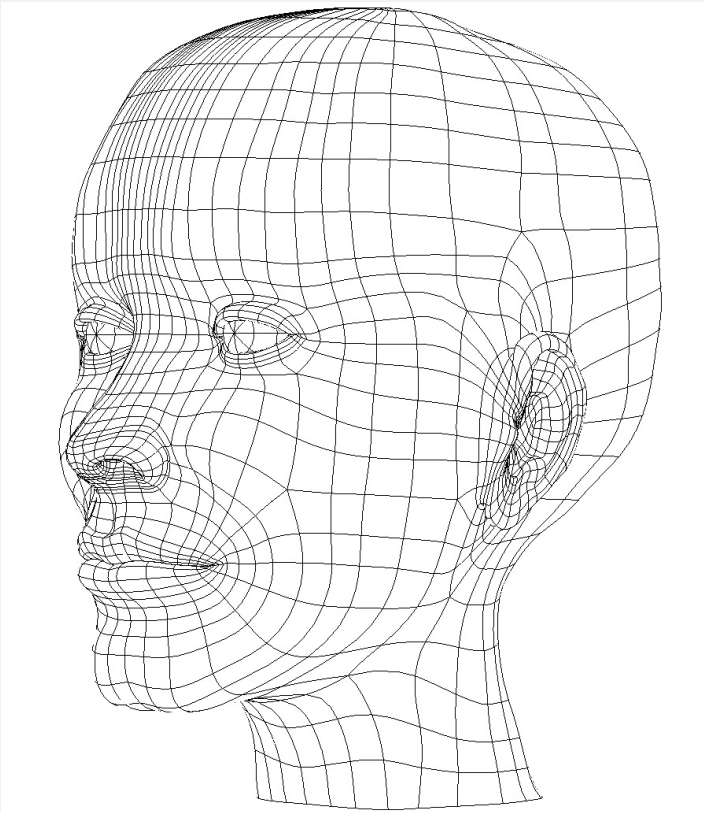

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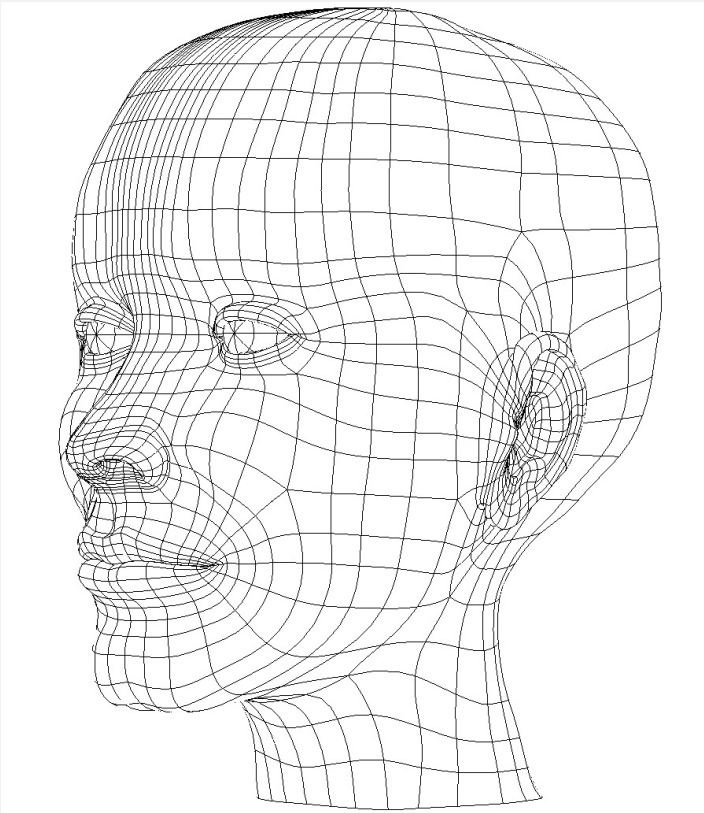

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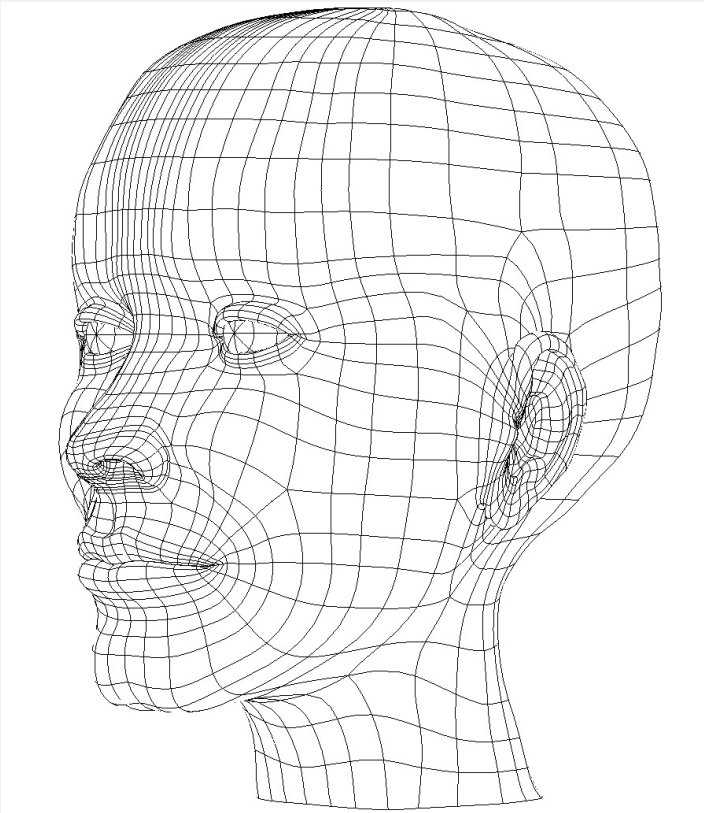

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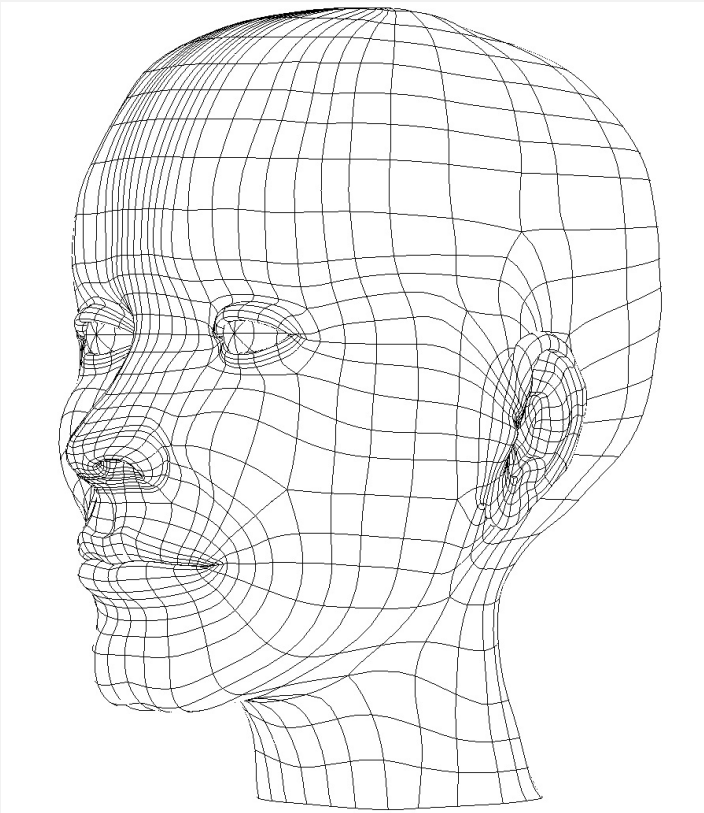

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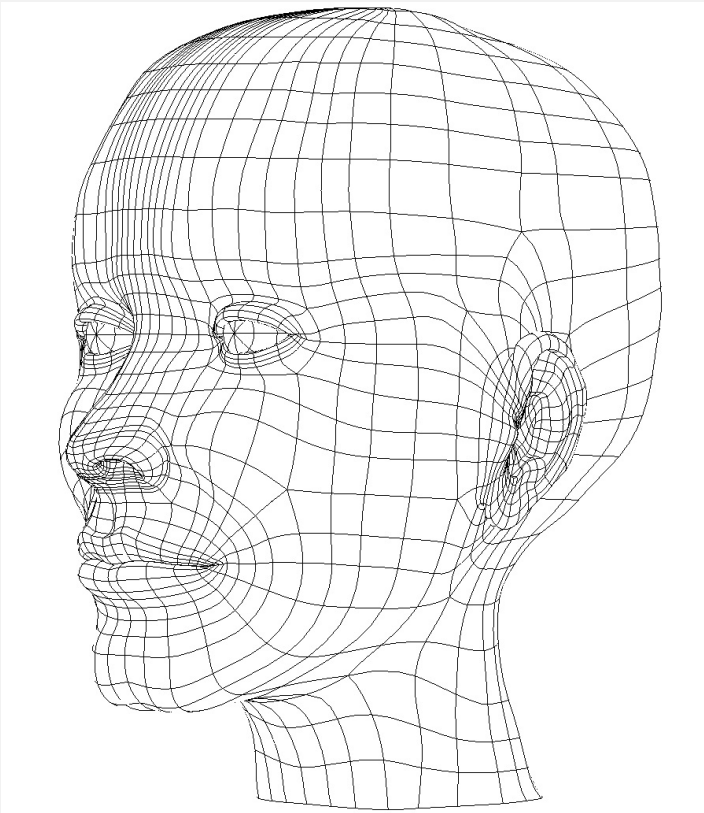

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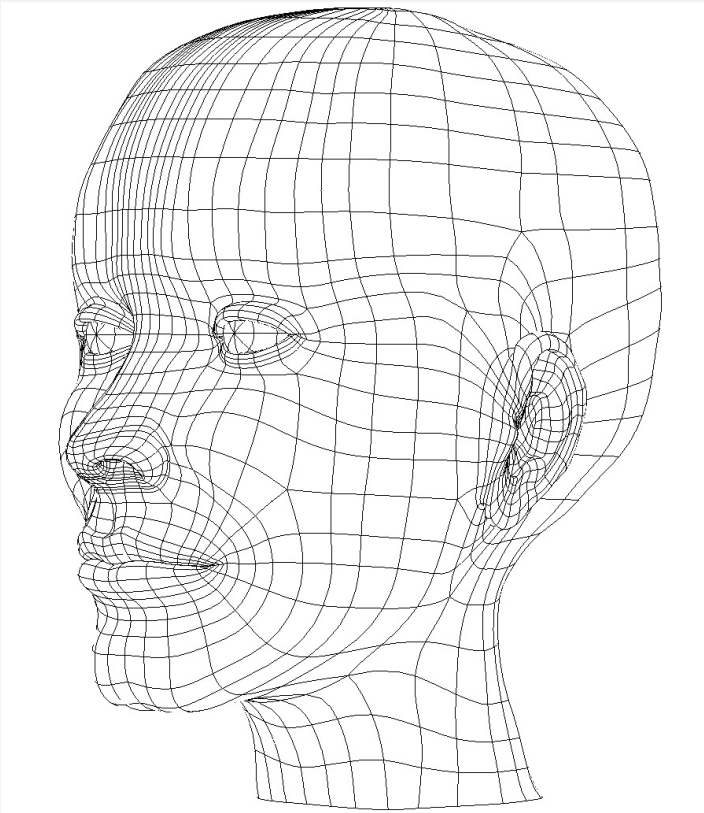

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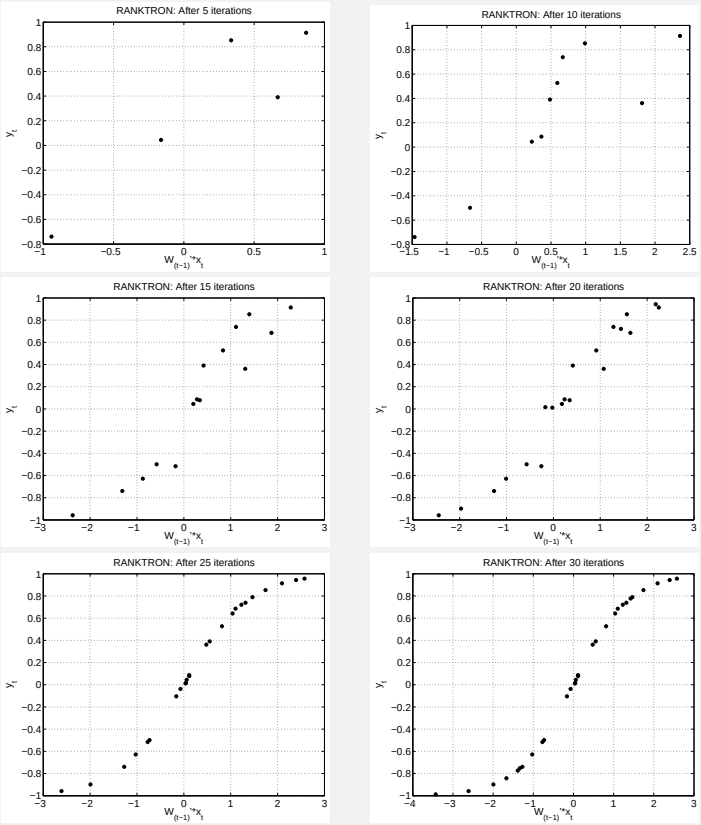

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- Tracking bounds.
- Example.

[Under Revision 2010-2011]

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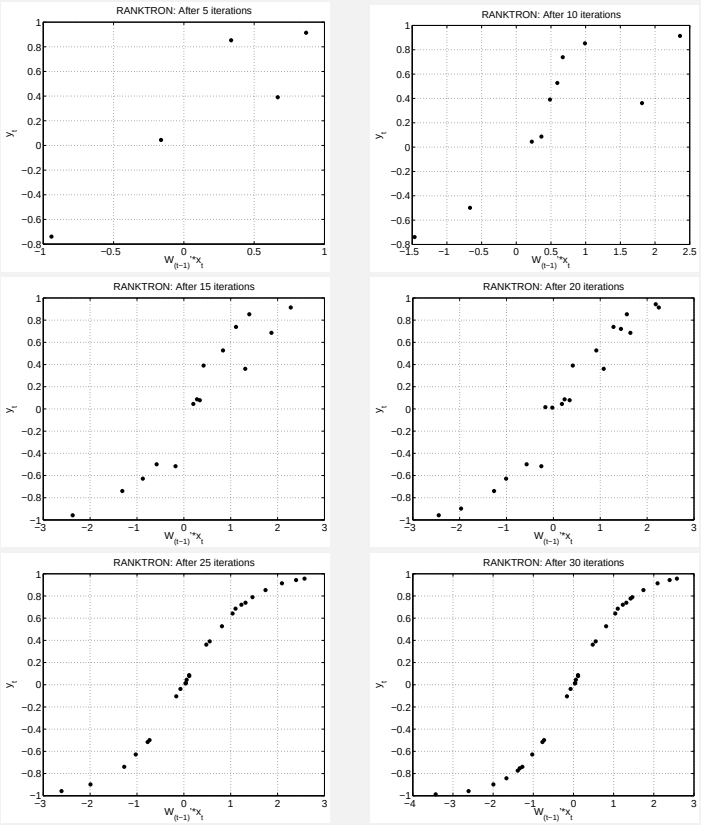

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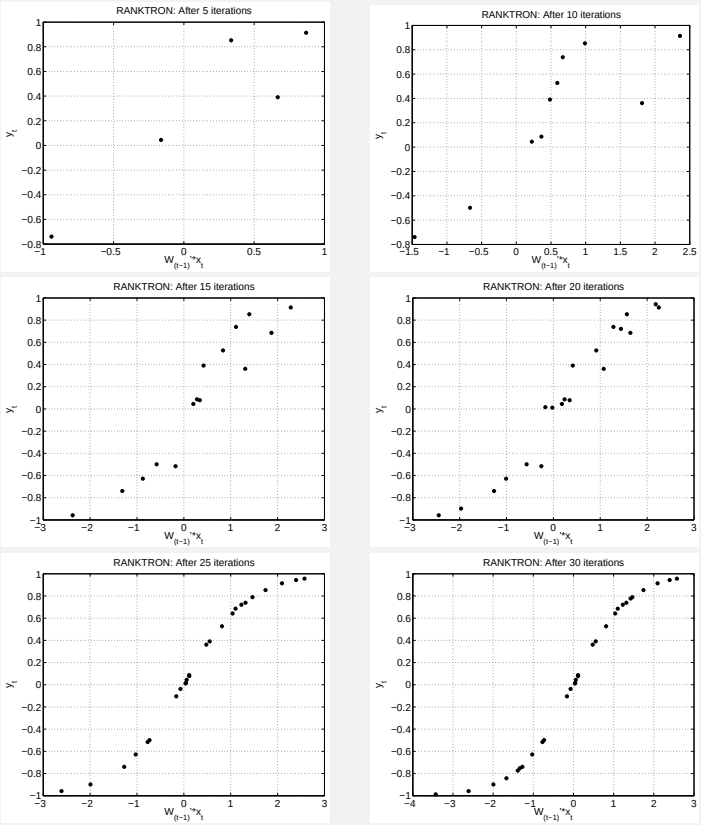

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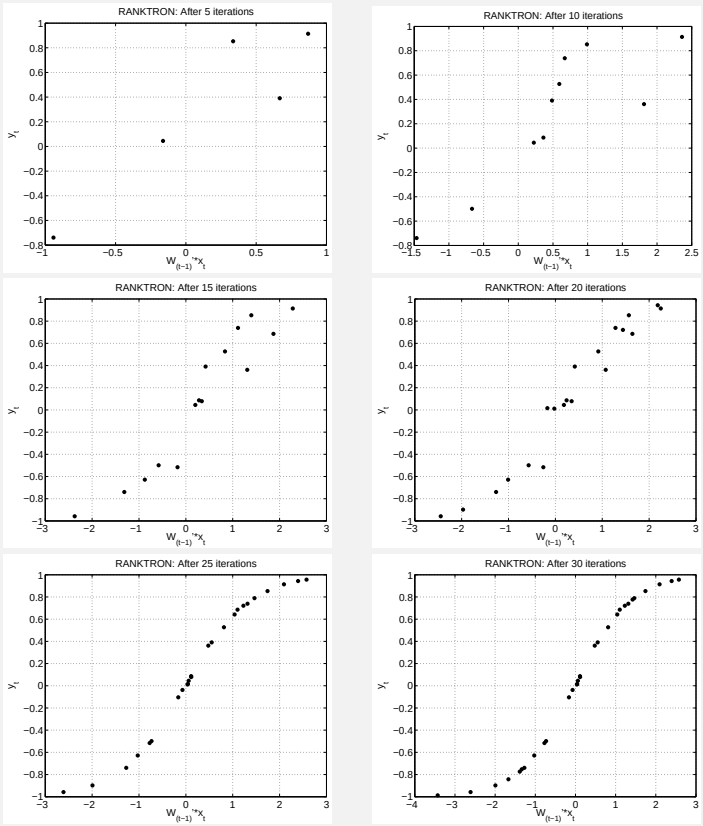

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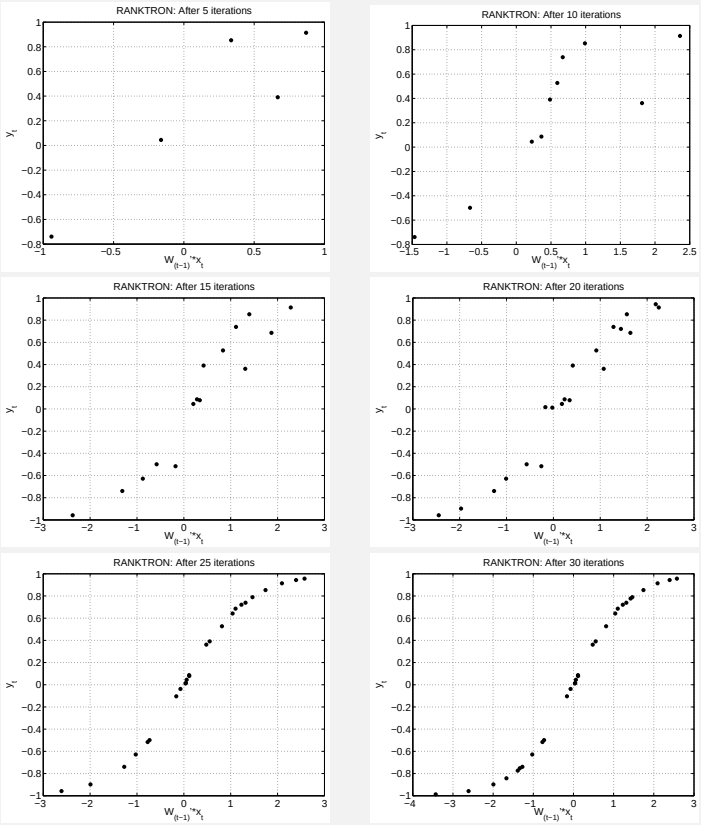

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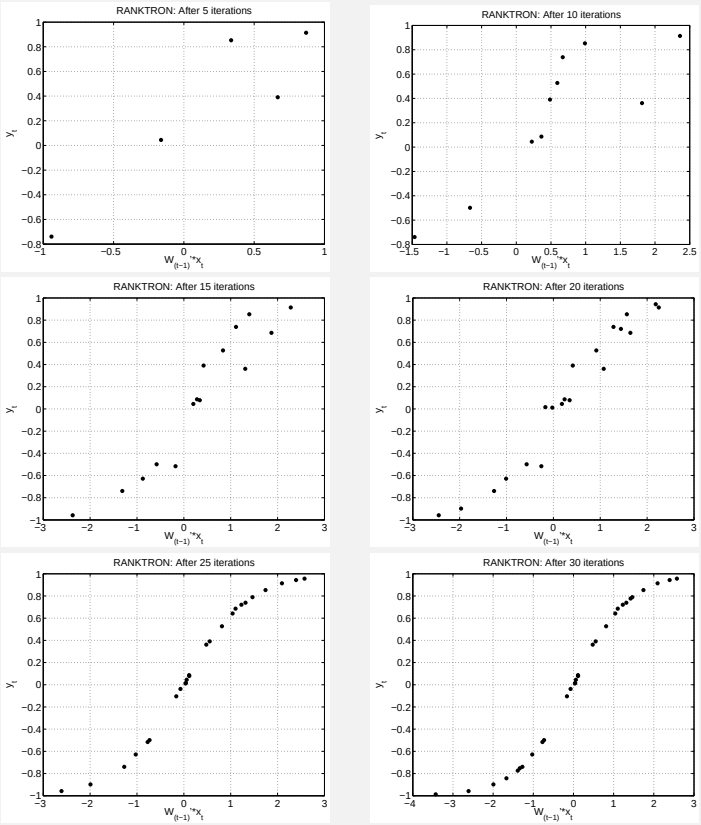

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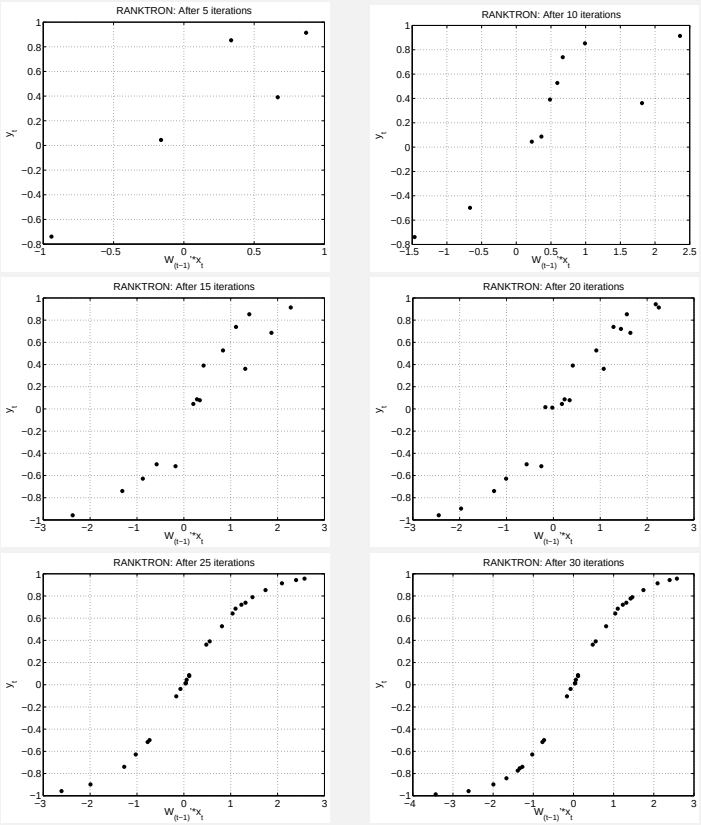

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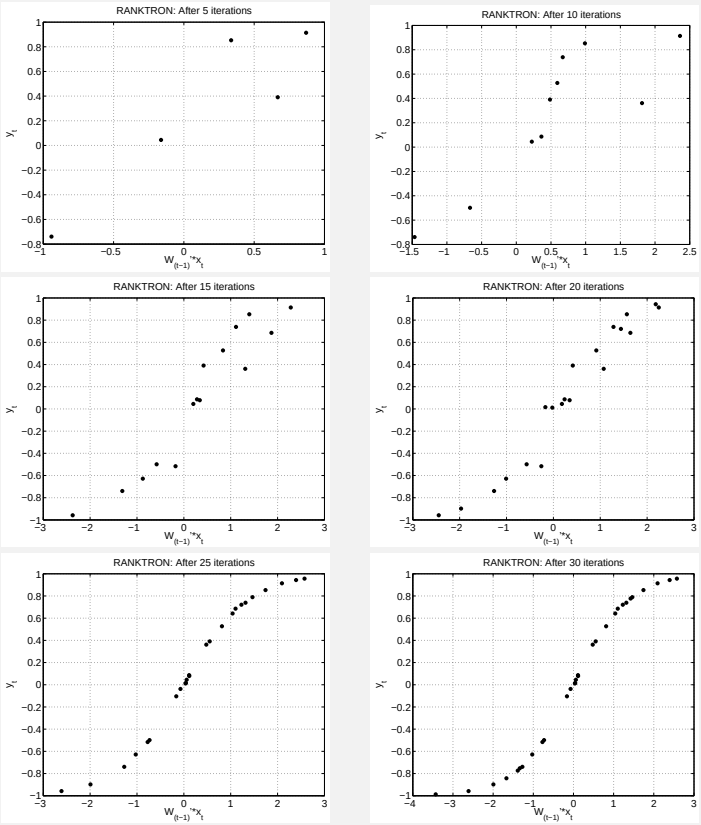

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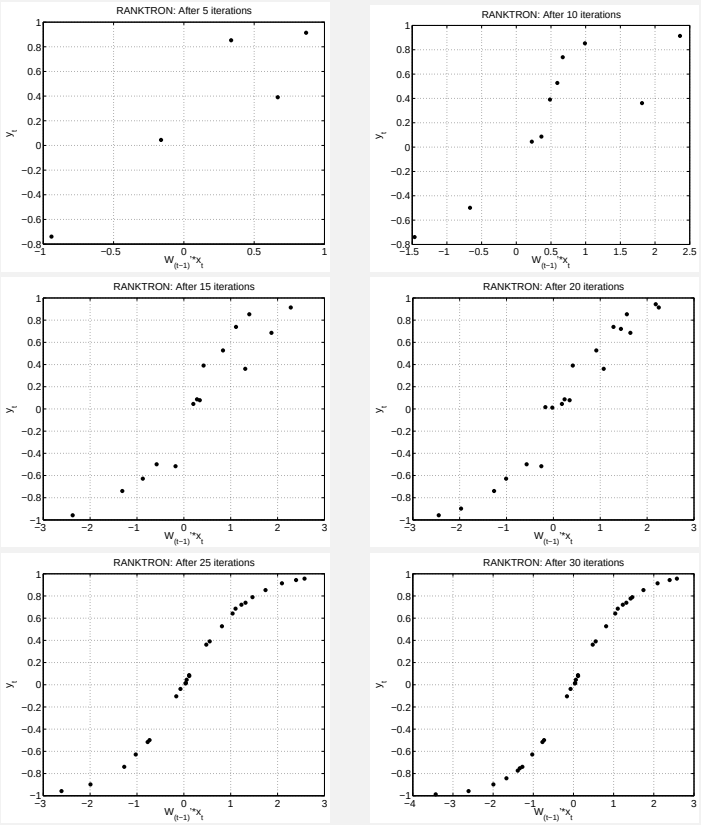

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Non-parameteric Control - NORTKNAR.

Q : Steer plant with $G(q^-1) = f_0(\mathbf{h}_0^T \mathbf{u}_t)$ to $\alpha \in \mathbb{R}$ by manipulating input $\{u_1, \ldots, u_t, \ldots\}$.

D : Totally Model-free: no need for first identifying $(f_0, \mathbf{h}_0)$ in *any form*.

- Initialize $t = 0$.
- Compute u_t.
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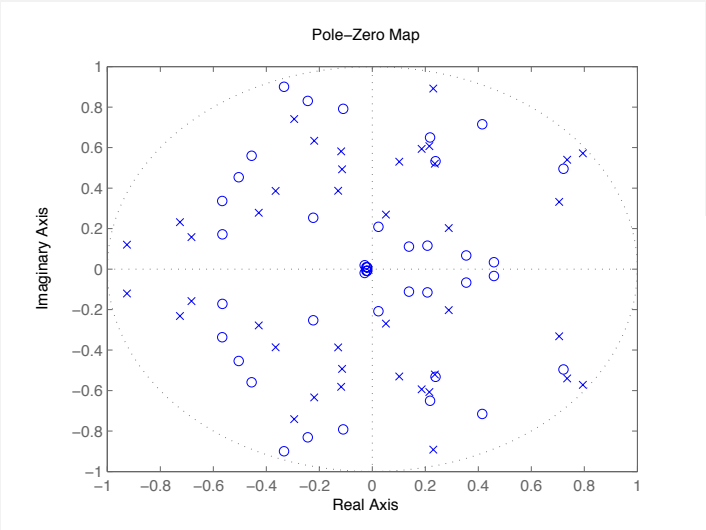

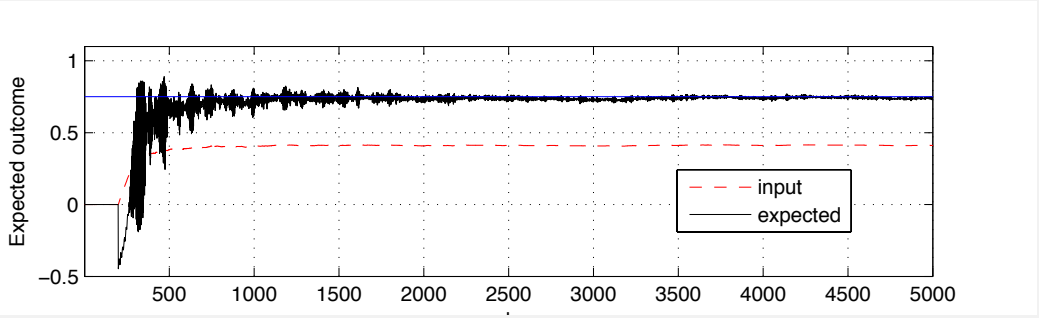

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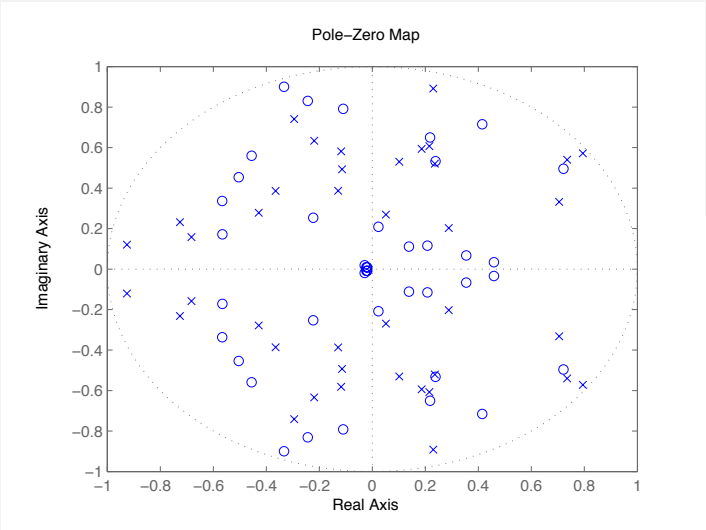

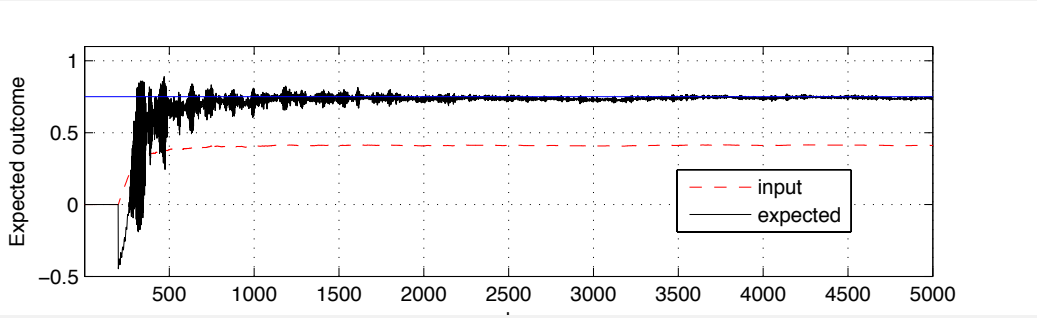

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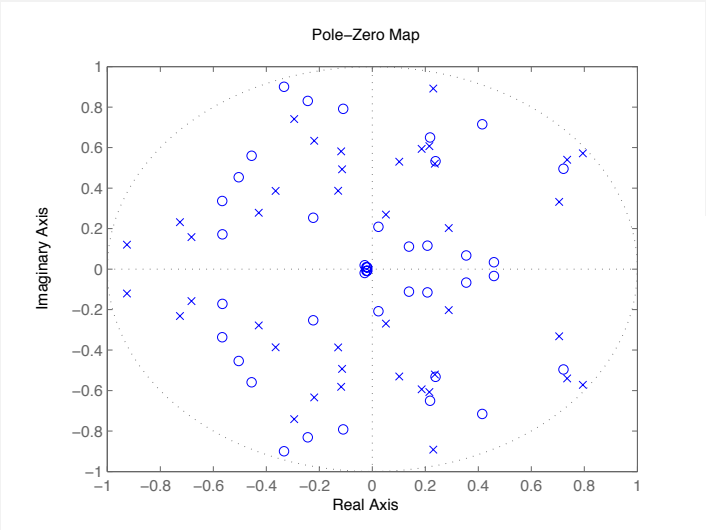

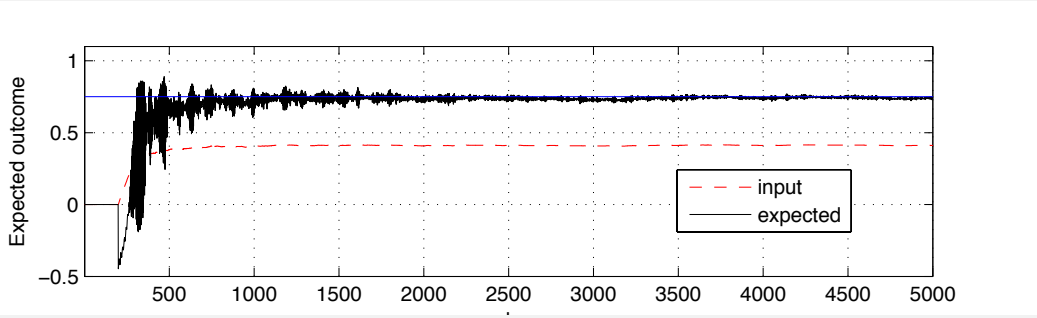

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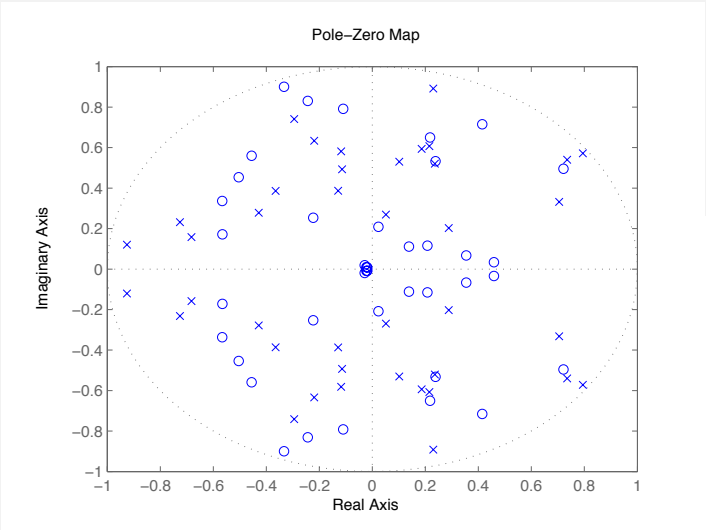

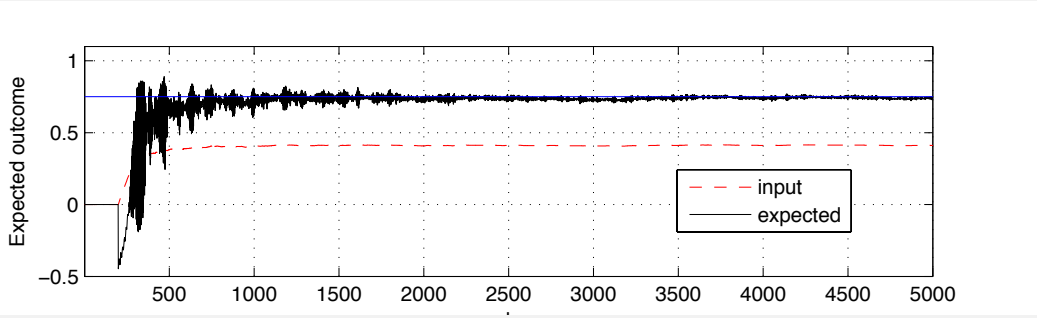

$$u_t = u_{t-1} + \eta_t(\alpha - y_t)$$

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  - $1_d^T \mathbf{h}_0 > 0$
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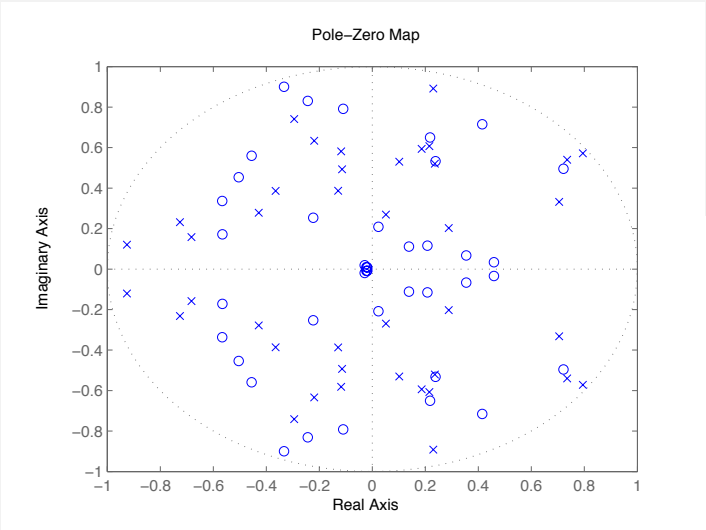

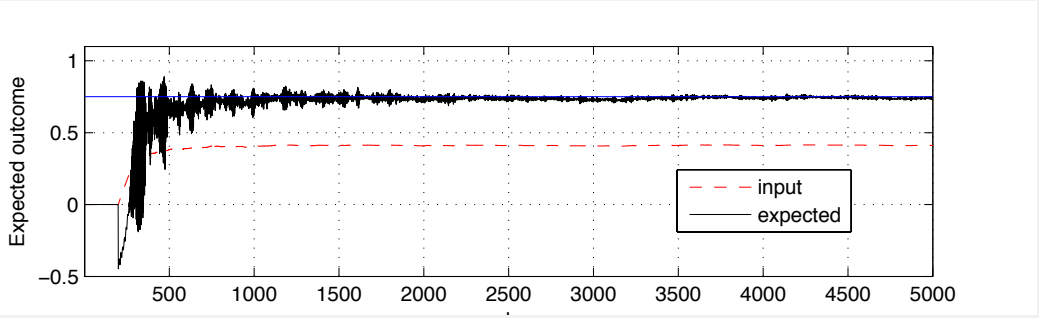

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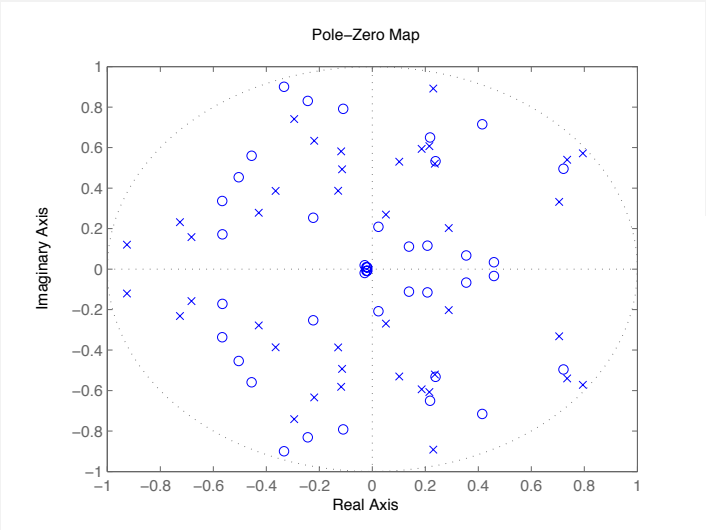

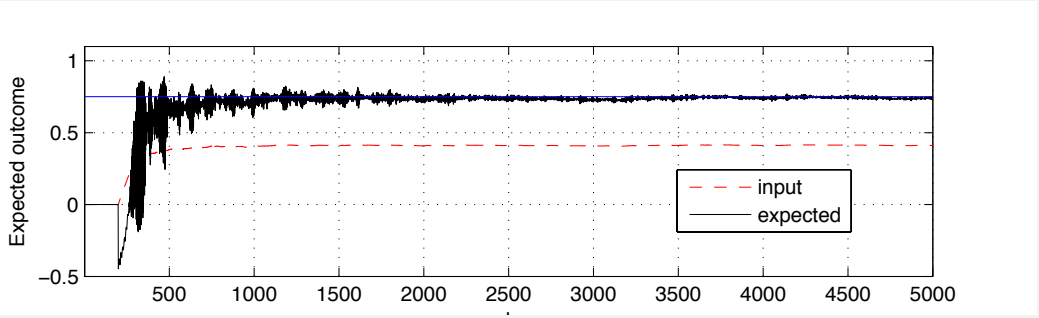

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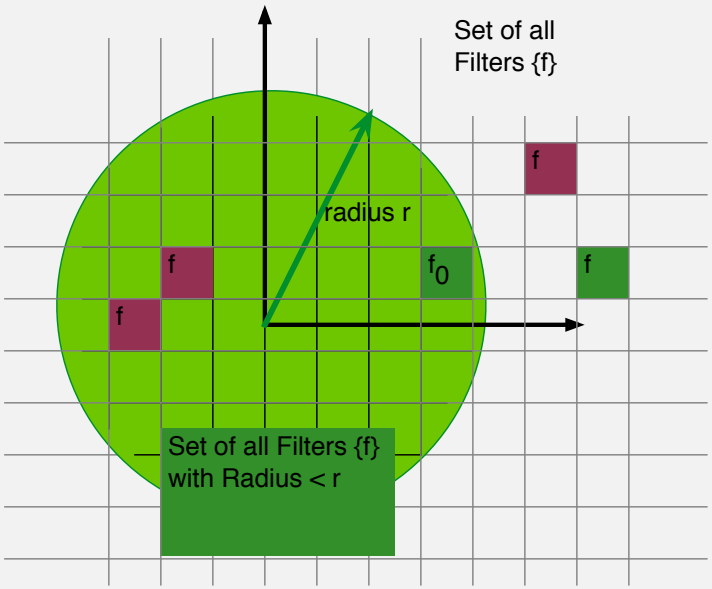
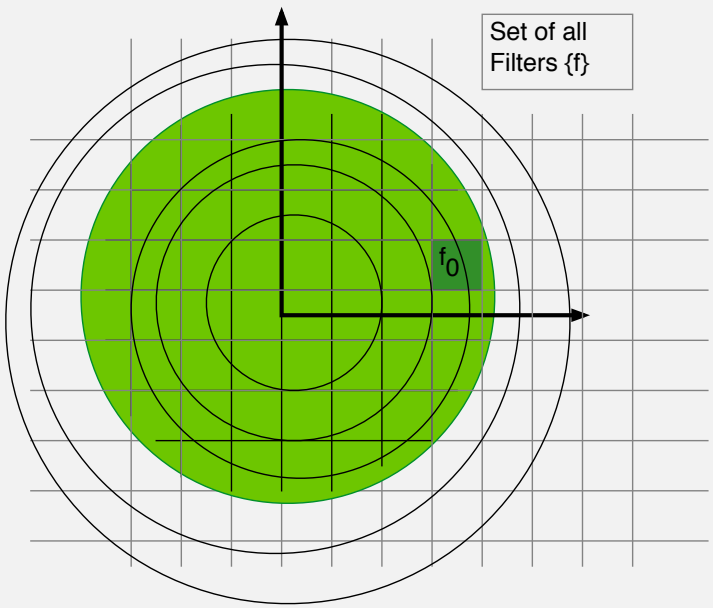
Complexity Control - Occam's razor revisited.



- Number of parameters?
- Number of free states?
- Desiderata:
  - Proper norm.
  - True $\mathbf{h}_0$ is low complexity.
  - Small number of f's with small complexity (covering).
  - $\mathbf{h} \sim \mathbf{h}'$, then order $\|\mathbf{h}\| < \|\mathbf{h}'\|$.
  - Only 1 $\mathbf{h}$ s.t. $\|\mathbf{h}\| = 0$.
  - f with small norm easily falsifiable.
  - Interpretation (sparseness, low rank, ...)
  - Convex.
- Examples.
  - $\|\mathbf{h}_0\|_2$
  - $\|\mathbf{h}_0\|_1$
  - $\|H(\mathbf{h}_0)\|_*$

Complexity Control - Occam's razor revisited.



"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience. AE33"

- Structure all fs in structured balls $\mathcal{H}_r$ as

$$\mathcal{H}_r = \{f : \ \|f\| \leq r\}$$

- Realizable case: true $f_0 \in \mathcal{H}_{\bar{r}}$.
- Rates $\frac{\log \bar{r}}{n}$.
- Probability error $\sim$ probability of not falsifying all fs less complex than f_0.
- Numerical trick $\rightarrow$ Principled approach.
- Limit distributions - Exponential?

Conclusions & Outlooks

- + Identification of Monotone Wiener Systems.
- + Slight extension of LTIs: exploring new ideas in a firm framework.
- + Non-parametric and Model-free Control.
- ? Noise.
- ? PE vs performance bounds.
- ? Derivation variance NORTKNAR.