Coherent Nonlinear Feedback and Applications to Quantum Optics on Chip

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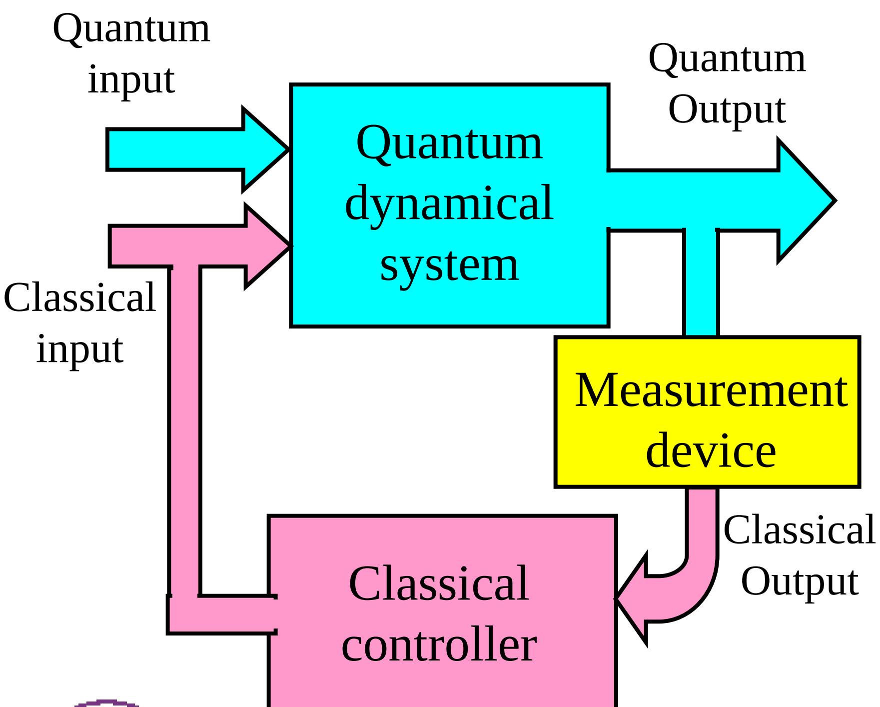
Outline

- Motivations
- Our main results
- Applications to on-chip quantum optics
- Conclusion

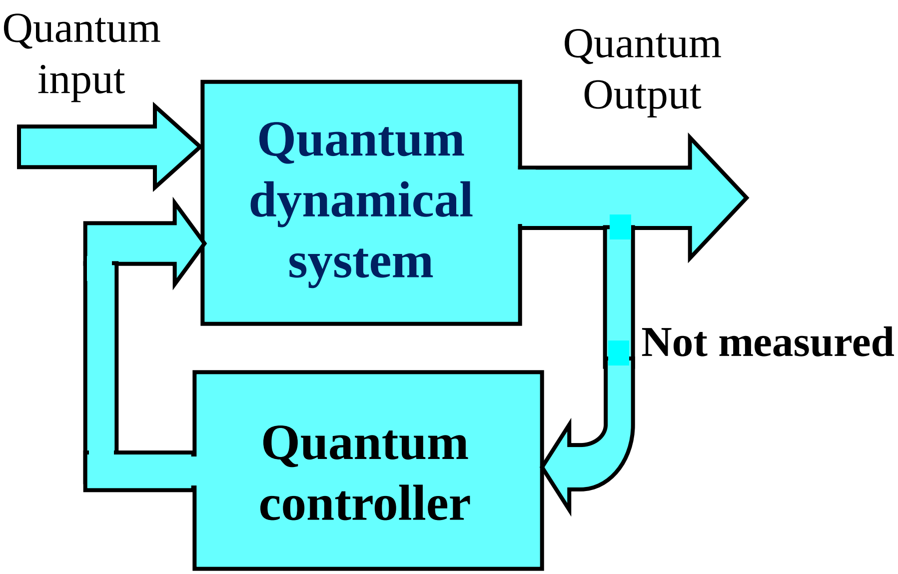
Feedback control of quantum system

- **Two different feedback methods to control quantum systems**

Classical feedback based on measurement



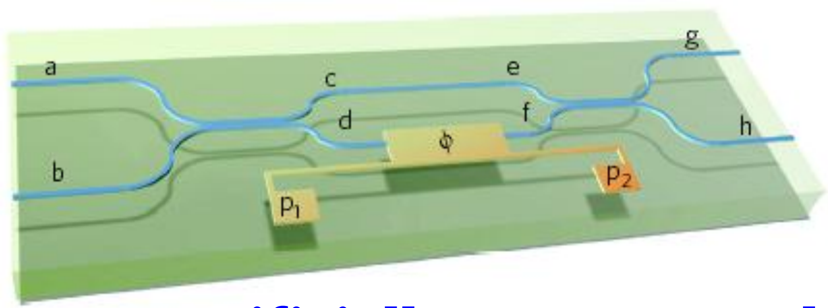
Quantum (coherent) feedback without measurement



Full quantum loop!

Motivations

- **A long-standing question in quantum control:**
- **Is there any problem that can be accomplished by quantum control, but not by classical control?**
- **Nonlinear quantum optics on chip**
- Natural nonlinearity is too weak to generate novel quantum optical phenomena. On-chip experiments are restricted in linear regime



- **Is there any way to artificially generate and enhance the desired nonlinearity, e.g., by feedback?**

Possible Solutions

- Can quantum nonlinearity be generated by classical (measurement-based) feedback control ?
  - The answer is no! (see our previous work in J. Zhang et al. Physical Review A 82, 022101 (2010));
  - Full quantum feedback loop (coherent feedback) has to be used !

Existing results in coherent feedback

- General theory of linear coherent feedback:

  - HP model: IEEE TAC, 54: 2530 (2009);

  - Quantum transfer function: IEEE TAC, 48: 2107 (2003)), Phys. Rev. A, 81: 023804 (2010).

- Nonlinear coherent feedback system has not been well (rarely) studied!

Our main results

- **Quantum amplification-feedback system: coherent feedback loop+quantum amplifier**

Feedback-induced nonlinear Hamiltonian

$$\dot{\rho} = -i\left[H_{eff}, \rho\right] + D\left[L_f\right]\rho + \left\{(N+1)D[L]\rho + ND\left[L^+\right]\rho\right.$$

$$\left. M^*\left(L\rho L - L^2\rho/2 - \rho L^2/2\right) + M\left(L^+\rho L^+ - L^{+2}\rho/2 - \rho L^{+2}/2\right)\right\}$$

Decoherence induced by the input vacuum field

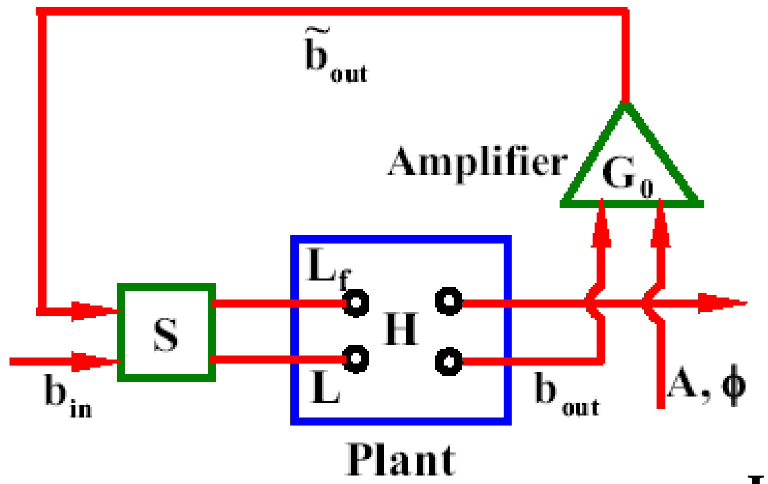
Decoherence induced by the quantum amplifier

$$D[L]\rho = L\rho L^+ - L^+L\rho/2 - \rho L^+L/2 \quad N = G_0 - 1, M = \sqrt{(G_0 - 1)G_0}$$

Cannot be obtained by HP model and quantum transfer function used for linear coherent feedback system!

Our main results

- **Amplifying quantum nonlinearity by quantum amplifier**



Enhanced by quantum amplifier

High-order terms induced by coherent feedback

$$H_{eff} = H + \tfrac{i}{2}\sqrt{G_0}\left[\left(L + L^+\right)S^+ L_f - L_f^+ S\left(L + L^+\right)\right] + \sqrt{G_0}\,A\left[(\cos\varphi)L + i(\sin\varphi)S^+ L_f + h.c.\right]$$

This term is introduced to cancel the linear term

Application to quantum optics on chip

- Generation of Kerr effect

$$H_{eff} = (\omega_a - \delta)a^+a + \underline{\chi(a^+a)^2}$$

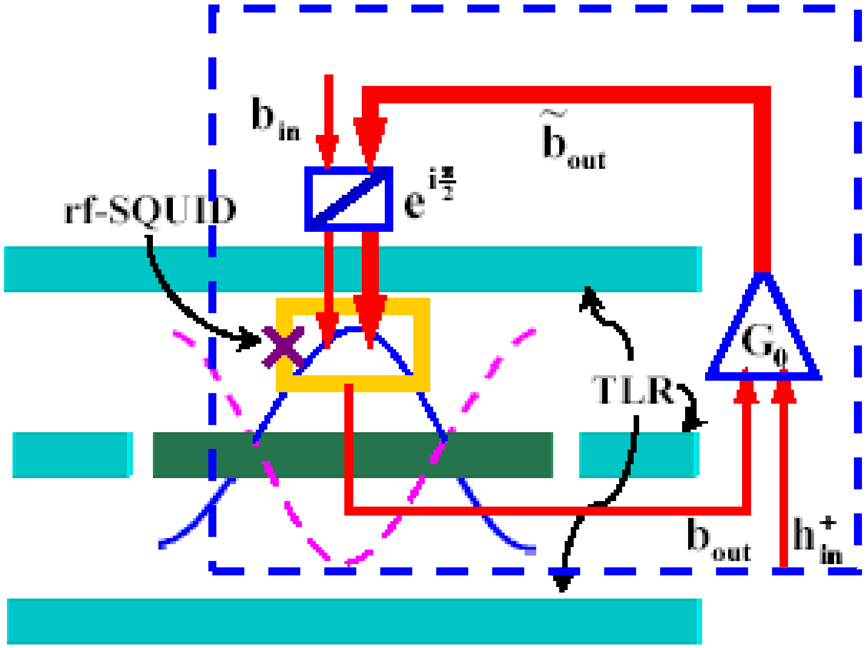
$$\delta = 2A\sqrt{G_0\gamma_a}$$

$$\chi = 2\sqrt{G_0\gamma_a}$$

induced Kerr term

10^4-10^5 stronger than its natural value!

Feedback loop



Potential applications: Schrödinger cat state preparation, quantum non-demolition measurement…

Application to quantum optics on chip

- Generation of more general fourth-order Hamiltonian

$$H_{eff} = \omega_a a^+ a + \sum_{k=1}^{4} \chi_k x_a^k,$$

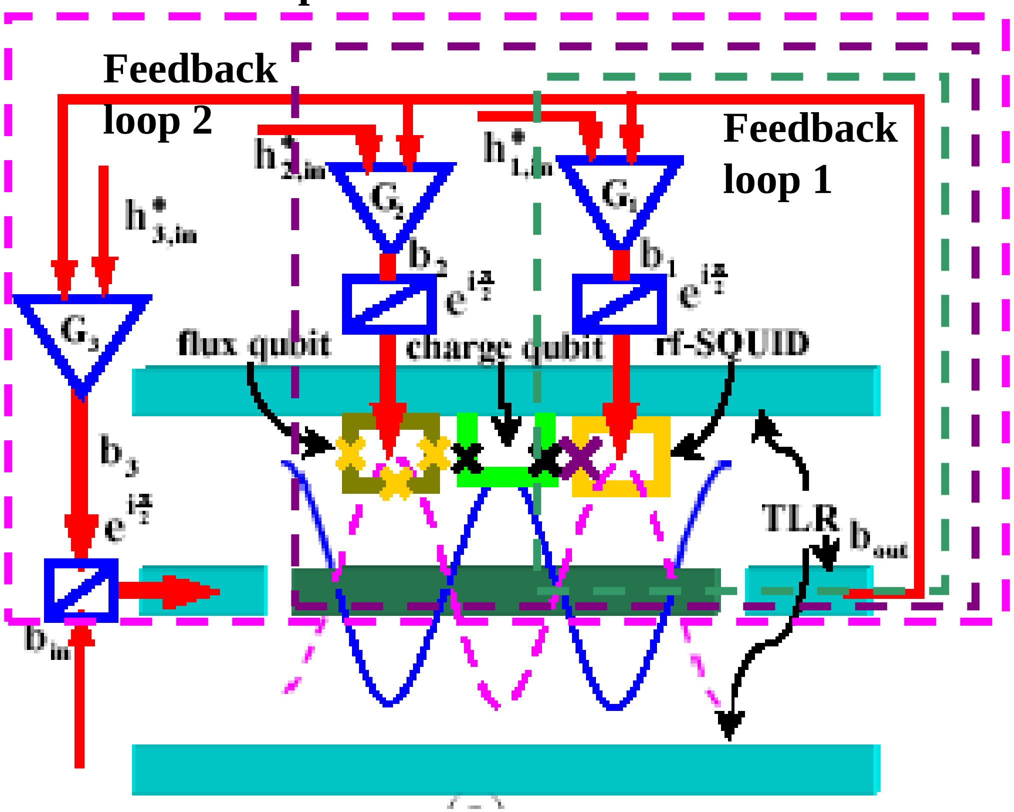
$$\chi_1 = A_4\sqrt{2\gamma},$$

$$\chi_2 = 4A_1\sqrt{G_1\gamma_1} - 2A_3\sqrt{G_3\gamma_3},$$

$$\chi_3 = 2\sqrt{G_3\gamma\gamma_3},$$

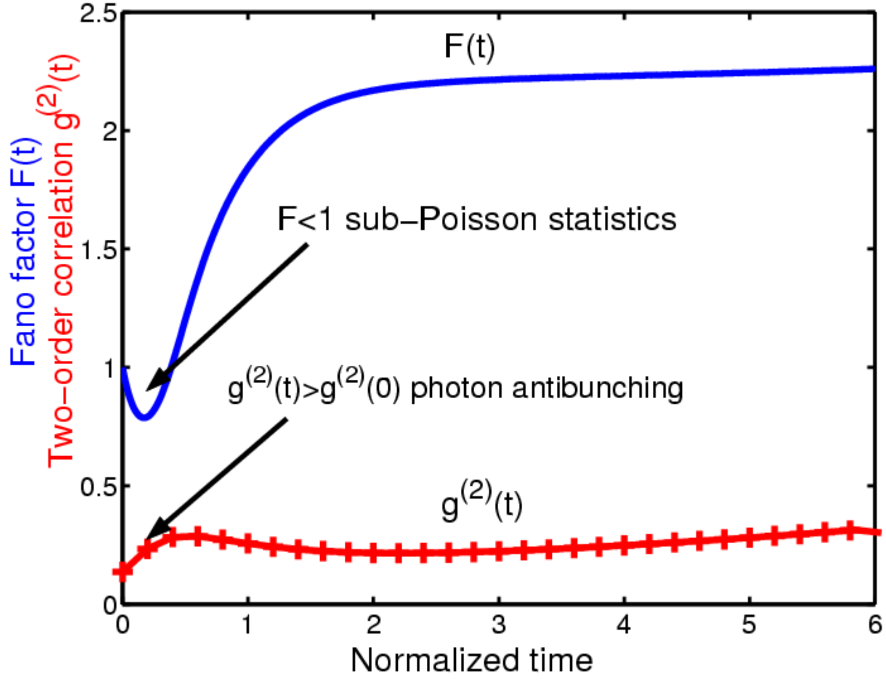
$$\chi_4 = 2\sqrt{G_1\gamma\gamma_1}$$

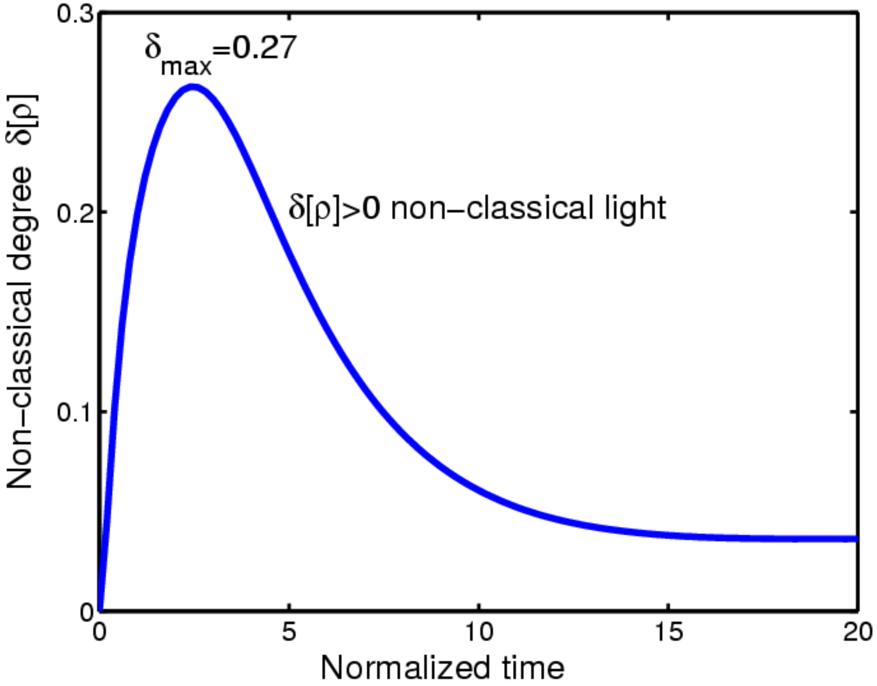
$\chi_1, \chi_2, \chi_3, \chi_4$ controllable



Application to quantum optics on chip

- Non-classical light obtained by the constructed nonlinearity





Conclusion

- Main contributions
  - Quantum feedback nonlinearization (QFN) can be realized by coherent feedback !
  - QFN has important applications in quantum optics on chip

- Open problems
  - Decoherence effect is also amplified. Quantum “PID” design?
  - Implementation complexity --- higher-order nonlinearity?

Thank you!